Fiduciary Obligations in the Presence of Multiple Classes of Stock

Sarath Sanga       Eric Talley*

September 6, 2018

Abstract

This paper develops a game theoretic framework to study the increasingly common conflict between common and preferred shareholders regarding whether to liquidate the firm or continue. In our model, common tend to inefficiently continue while preferred tend to inefficiently liquidate. Taking a cue from recent case law, we explore whether it is possible to use damages (for either “wrongful exit” or “wrongful continuation”) to align the interests of common and preferred in maximizing firm value. We show that there always exists an efficient damages rule if the interests of preferred shareholders control the exit/continue decision. When common control the decision, however, an efficient damages regime may either fail to exist or may require supracompensatory damages. Our framework also suggests that ex ante contracting need not give rise to an efficient damages rule, particularly if investment capital is relatively scarce. Our findings have implications for the ongoing debate about how to assign fiduciary duties and rights within privately held firms with multiple classes of stock.

Keywords Preferred stock, common stock, capital structure, fiduciary duties, optimal contracting

JEL G30, G34

*Sanga: sanga@northwestern.edu. Talley: et2520@columbia.edu. PRELIMINARY AND INCOMPLETE DISCUSSION DRAFT: DO NOT QUOTE OR CITE WITHOUT PERMISSION
# Contents

1 Introduction 3

2 Setup of the Model 11

3 The Conflict Between Common and Preferred 13

4 Optimal Contracting Under a Common Chooses Rule (“cc”) 15
   4.1 No Liability ................................. 15
      4.1.1 Socially Efficient Contract .................. 16
      4.1.2 Privately Optimal Contract ................. 16
   4.2 Liability for “Wrongful Continuation.” ............. 18
      4.2.1 Socially Efficient Contract .................. 19
      4.2.2 Privately Optimal Contract ................. 20

5 Optimal Contracting Under a Preferred Chooses Rule (“pc”) 23
   5.1 No Liability .................................. 23
      5.1.1 Socially Efficient Contract .................. 24
      5.1.2 Privately Optimal Contract ................. 24
   5.2 Liability for “Wrongful Exit” ........................ 25
      5.2.1 Socially Efficient Contract .................. 27
      5.2.2 Privately Optimal Contract ................. 28

6 Extensions 30

7 Conclusion 31
1 Introduction

Within US company law, corporate fiduciaries are frequently said to owe legal obligations “to the corporation and its shareholders.”\(^1\) This compound formulation is shorthand for the proposition that fiduciaries—while technically not trustees—are obligated to manage the corporation for the ultimate benefit of its residual claimants (i.e., its shareholders).\(^2\) The rhetorically bundled nature of this fiduciary obligation—running simultaneously to the corporation and its shareholders—is not overly problematic so long as shareholders share a single, unified interest. But what happens when they disagree? How are corporate fiduciaries to choose among obligations owed to respective shareholder constituencies?

In public companies, these questions animate a heated debate between various types of shareholders that are (purportedly) attracted to “short term” versus “long term” investment horizons.\(^3\) In these debates, shareholders tend to hold the same securities but differ in their time preferences or strategic commitments. Even this minor difference can make discharging fiduciary obligations an onerous if not impossible task, as directors must either (a) choose a side or (b) try to argue (or at least pretend) that a proposed business strategy is in everyone’s interest.

Within privately held firms, and particularly those in the venture capital and private equity space, the intractability of this inter-shareholder conflict is even more pronounced: for it is effectively baked into the capital structure of the firm. Consider, for example, a venture capital (VC) investor that has

---

\(^1\) North American Catholic Education Programming Foundation, Inc. v. Gheewalla, 930 A.2d 92, 99 (Del. 2007).

\(^2\) “The directors of Delaware corporations have the legal responsibility to manage the business of a corporation for the benefit of its shareholders owners.” North American Catholic Education Programming Foundation, Inc. v. Gheewalla, 930 A.2d 92, 101 (Del. 2007) (quotations omitted).

\(^3\) See, e.g., Barzuza and Talley (2016). Both scholarship and case law provide many other instances in which shareholders have conflicting interests. For example, shareholders have conflicting interests when they transact with the company (Kahn v. M & F Worldwide Corp., 88 A.3d 635 (Del. 2014)) when they own (or are) competing businesses (Gilo (2000); Azar, Schmalz and Tecu (2015); Sanga (2018); Sinclair Oil Corp. v. Levien, 280 A.2d 717 (Del. 1971)), when their ownership is intermediated by others (Bartlett (2006)), when they possess disproportionate voting power (Masulis, Wang and Xie (2009); Bebchuk, Kraakman and Triantis (2000)), or when they promote opposing social objectives (Webber (2018)).
injected $100 in a startup company controlled by its Founder. As is typical in such arrangements, the VC’s investment takes the form of convertible preferred stock, while the founder retains common shares. Suppose that in the event of a liquidation or sale of the firm, the Preferred Stock has a liquidation preference of $100, above which the Founder becomes the sole residual claimant up to some “catch up” point (say, $200). Beyond the catch up point, the preferred stockholders would find it optimal to convert into common, splitting 50/50 with the Founder the total liquidation proceeds. At the time of the VC’s investment, the Founder and VC agree that the board will consist of the Founder and two designees of the VC.

Now imagine that after several years, and through the efforts of the VC’s director-designees, the company receives an acquisition offer of $100. The Founder and the two VC directors all believe that if they continue the firm until it runs out of capital, there is a 50% chance the firm will be acquired for $300 and a 50% chance that it will be liquidated for $0. Given the VC’s liquidation preference, the director-designees prefer to sell the company today because the VC would receive $100 with certainty as opposed to a 50% chance of receiving $150 and a 50% chance of $0. The Founder, in contrast, prefers to continue because that would yield a 50% chance of receiving $150 as opposed to $0 with certainty. What will the directors decide? Can the VC’s directors vote the preferred shareholder’s interest with a clear conscience and without violating their fiduciary duties?

If this legal conundrum sounds familiar, it is because we have been here before—at least approximately. Over a quarter-century ago, a similar set of disputes erupted between shareholders on the one hand and creditors on the other. These conflicts concerned corporate actions that were taken while the firm was in the so-called “zone of insolvency.” For firms in financial distress, boards must often decide between actions that benefit creditors (e.g., liquidating a firm with little or no payout to stockholders) and actions that benefit stockholders (e.g., continuing a firm but at the risk of losing the company’s remaining assets). The usual question in these shareholder-creditor disputes was whether directors should be obliged to maximize a firm’s total value, or merely its shareholders’ residual claim. And in a famous footnote to the 1991 Chancery Court case of Credit Lyonnais Bank Nederland, N.V. v. Pathe Communications Corp, then-Chancellor William Chandler suggested the former: In non-binding obiter dictum, he mused that within the zone of insolvency, directors’ fiduciary obligations run to the “community of interests that the corporation represents,” even if the corporate actions necessary to
advance these interests are inconsistent with actions that maximize shareholder returns.

Significant confusion ensued for many years afterward, as courts grappled with the meaning of this language and creditors brought claims against distressed firms, challenging corporate decisions that benefited shareholders at the expense of creditors. More than 15 years passed before the Delaware Supreme Court finally put an end to the debate (or so it thought). In *North American Catholic Educational Programming Foundation, Inc. v. Gheewalla*, the Delaware Supreme Court reversed course and held that creditors have no rights under fiduciary law so long as the firm remains solvent:

> When a solvent corporation is navigating in the zone of insolvency, the focus for Delaware directors does not change: directors must continue to discharge their fiduciary duties to the corporation and its shareholders by exercising their business judgment in the best interests of the corporation for the benefit of its shareholder owners.4

*Gheewalla*, however, resolved only conflicts between debt and equity. It was silent about how to confront conflicts among equity holders. There, in contrast, Delaware has continued to struggle to resolve conflicts like the VC vignette presented above; and a series of recent cases have presented what has become a familiar fact pattern.5 In each of these cases, a venture capital firm holding preferred stock seeks to force an exit either by exercising its majority voting rights or by visiting a capital shortage on the firm (e.g., by redeeming its preferred shares).6 The corporate documents memorializing common and preferred shareholders’ rights often provide some guidance, yet they do not completely specify the steps that directors may (or must) take to force an

---


6Venture capital firms almost always own preferred stock (Kaplan and Strömberg, 2003; Gilson and Schizer, 2003). On the legal conflicts that obtain, see, e.g., Bratton (2002); Fried and Ganor (2006). Our focus on the preferred-common conflict abstracts from important conflicts among preferred stockholders that arise out of the staged and syndicated nature of venture capital investments (Bartlett, 2006).
These cases thus expose a fundamental indeterminacy in directors’ and officers’ fiduciary obligations, as courts are forced to search within fiduciary law to both fill in these gaps and to ensure that the resulting arrangement conforms with non-waivable fiduciary duties of loyalty to the corporation.

The solution that Delaware courts have begun to gravitate toward is to require directors to advance the interests of “stockholders in the aggregate . . . without regard to any special rights” possessed by preferred shareholders. In principle, this requires directors to first identify preferred and common’s shared interest, and then to advance that joint interest. But this admonition is of little help when—as in the actual cases—preferred and common adamantly disagree over which course of action in fact promotes the position of “shareholders in the aggregate.” If we add to this the profound uncertainty inherent in comparing the value of a firm’s current assets against its value as a going concern, then the obligation to promote “shareholders in the aggregate” in the face of irreconcilable inter-shareholder conflict seems little more than a jurisprudential wish that the conflict did not exist.

In the recent case of Hsu v. ODN, Vice Chancellor Laster suggested a possible way out of this fiduciary conundrum. He opined that instead of searching within the realm of fiduciary law, a court may find a clearer solution outside of it, in contract law: specifically within the doctrine of “efficient breach.” Laster’s proposed solution would require the board to act in the best interests of common shareholders, as the most residual of claimants, a remit that would possibly require breaching several of the special rights that preferred might otherwise have to control the decision. But in the process, preferred shareholders would also be entitled to damages against the firm for something akin to “wrongful continuation,” that is, for cases in which the firm continues operations over the objection of preferred shareholders. If these damages were properly calibrated, he opined, this would in turn cause

---

7For example, in the ODN case, redemptions could only be made out of funds that are “legally available,” and such funds could only be generated by “reasonable actions (as determined by the [ODN’s] Board of Directors in good faith and consistent with its fiduciary duties).” ODN at 4. Neither term was precisely defined.

8Frederick Hsu Living Trust v. ODN Holding Corp., No. CV 12108-VCL, 2017 WL 1437308, at 17 (Del. Ch. April 14, 2017), as corrected (April 24, 2017). Accord Trados Inc. Shareholder Litigation, 73 A.3d 17, 39-40 (Del. Ch. 2013) (“A board does not owe fiduciary duties to preferred stockholders when considering whether or not to take corporate action that might trigger or circumvent the preferred stockholders’ contractual rights. Preferred stockholders are owed fiduciary duties only when they do not invoke their special contractual rights and rely on a right shared equally with the common stock.”)
directors to internalize the conflict between common and preferred.\textsuperscript{9}

Although not explicitly floated by Vice Chancellor Laster, his opinion also suggests an alternative doctrinal possibility that comes from the opposite direction: preferred shareholders might be accorded primacy over the exit decision, with common shareholders enjoying the right to recover contract-like damages against the firm for “wrongful liquidation.” Both Laster’s approach and this one are premised in contract law principles, and both appear promising; but neither has been thoroughly explored in the academic literature.

In this paper, we seek to start filling this void. In the process, we also hope to provide some navigational landmarks for courts as they develop the law in this area. Our contribution is predominantly theoretical, as we develop a game theoretic capital structure model of a privately-held firm having both common and preferred shares. Our analysis draws on the standard Black-Scholes-Merton framework, in which respective shareholders’ cash flow positions are represented as a series of options.\textsuperscript{10} To this we layer on control rights over whether to exit early (through an outside acquisition offer), or to continue the firm under the status quo to a later terminal date. Our model explicitly accounts for the incentives that must be provided both (a) to induce preferred shareholders invest and (b) to induce common (typically the founder and key employees) to exert productive effort. We examine the efficiency consequences of assigning control over the exit/continue decision to preferred versus common shareholders—an exercise we conduct first in the absence of any additional damages exposure, and in the presence of damages. With regard to the latter, we ask whether there exist liability rules that induce the party controlling the decision (preferred or common, as the case may be) to make jointly optimal decisions about whether to exit or continue, and if so, what the contours of such optimal damages are. Finally, we layer on an initial contracting stage to gain traction on whether the parties themselves would tend to gravitate to efficient contractual terms through ex ante bargaining.

Our analysis delivers some familiar insights, as well as some others that we think are both less obvious and potentially important to this ongoing jurisprudential puzzle. Consistent with intuition, preferred shareholders in our model tend to be too eager to exit: They are willing to sell even when the bid falls short of the going concern value of the firm as an enterprise.

\textsuperscript{9}Hsu v. ODN Holding, at 48
\textsuperscript{10}See Merton (1973); Black and Scholes (1973).
On the other hand, common are too reluctant to exit. They tend to favor continuation even for some bids that clearly exceed going concern value. This problem of skewed incentives, moreover, is quite general and endemic to standard VC/PE capital structures—at least in the absence of a credible liability threat.

Somewhat less intuitively, our model reveals conditions under which there exists a liability rule that aligns the controlling party’s choice with joint value maximization. When preferred control the liquidation decision, we demonstrate that there always exists a liability regime whereby common can extract damages for “wrongful” exit decisions by the preferred. The simplest of such regimes imposes strict liability on the preferred for all early exits, with damages equal to the common’s “expectation.” In an option-theory framework, this expectation corresponds to the value of the common’s call-option position in the company’s cash flows (offset by any revenues received by the common as a result of the exit). In contrast, when common controls the exit decision, we show that an efficient liability regime need not always exist, particularly when the preferred shareholders’ liquidation preference is large relative to the total value of the firm. Moreover, even when an efficient liability rule exists under a common chooses regime (i.e., when the liquidation preference is moderate or small), the damages associated with “wrongful continuation” bear no resemblance to expectation damages, and may even appear punitive in nature.

This asymmetry between preferred versus common liability rules comes from the timing and seniority of wrongful continuation damages: because common shareholders are liquidity constrained, recovery for the preferreds must come out of the future “terminal” value of the company. Should that terminal value be small (or even zero), the there is nothing to pay the preferreds. Consequently, when common controls, the preferreds can be compensated only when the firm does extremely well by continuing, necessitating a supracompensatory award in those contingencies.

Backing up a stage, we also demonstrate that parties may not have sufficient incentives ex ante to contract for jointly efficient terms. That is, the private agreements between preferred and common shareholders need not (and in some cases will not) allocate control rights and liability exposure in a way that maximizes firm value. In our model, investment capital is assumed to be relatively scarce and preferred shareholders therefore possess significant bargaining power over ex ante contractual terms. We show that in such settings, preferred will systematically move away from the most efficient
control/liability regime (on the margin) in order to transfer economic rents to themselves.

Our analysis delivers several practical insights for how the law regulates conflicts between common and preferred shareholders. First, there may be very little courts can do to incentivize efficient conduct when common shareholders enjoy primacy over the exit/continue decision. As noted above, an efficient liability regime simply may not exist when liquidation preferences grow large. And even when one exits, optimal damages may be supra-compensatory, thus resembling punitive damages. As is well known, Delaware courts generally may not award punitive damages without prior statutory authorization.\footnote{See, e.g., Moore v. Graybeal, No. 340, 1988, slip op. at 3 (Walsh, J.) (Del. Oct. 28, 1988) (ORDER), disposition reported at 550 A.2d 35 (Del. 1988) (TABLE); Kaye v. Pantone, Inc., 395 A.2d 369, 372 (Del. Ch. 1978); Beals v. Washington International, Inc., 386 A.2d 1156, 1159 (Del. Ch. 1978).}

Our analysis also reveals that when the interests of preferred shareholders dictate the exit/continue decision, there generally does exist an efficient damages regime. In this regime, common recovers a strictly positive damage award pegged against the option value of its position as of the date of exit. This observation suggests that even if the price paid at exit is below the preferreds’ liquidation value, common should nonetheless be entitled to a priority compensatory claim (qua judgment creditor) on the proceeds. The fact that the common’s shares are technically “under water” does not negate this point.\footnote{This intuition is potentially in tension with the decision in \textit{Trados}, where Vice Chancellor Laster awarded nothing to the common shareholders, concluding that they did not have a reasonable prospect for any upside if the firm continued. The ultimate disposition in \textit{Trados} is consistent with our model only to the extent that common shareholders held a deep out-of-the-money call option within the firm’s capital structure. Otherwise, our model suggests that common should receive a positive option value for their claim under an optimal rule.}

Finally, our analysis suggests that efficiency-minded courts may well be justified in assessing efficient damages rules that are mandatory (rather than default) by nature. When investment capital is scarce, the parties’ privately optimal contract likely diverges from the socially efficient one. By committing the parties to an efficient liability rule for exit/continuation decisions, and further by not allowing them to change it, courts can induce contracting parties to move closer to efficient contracting.

Before proceeding, several caveats to our arguments deserve specific men-
tion. First, as alluded to above, our analysis hinges on whether common or preferred enjoy control over the exit/continue decision. Such control could be an artifact of the governance structure (such as enjoying more votes, more board seats, hefty redemption rights, drag-along provisions, and the like); or, it may be the product of legally imposed duties (such as duties requiring fiduciaries to focus solely on the payoff of the common, or solely the preferred). Our model permits an analysis of efficient damages regimes regardless of the authority that vests control in one group’s hands.

Second, in order to focus on the inter-shareholder conflict, our analysis simplifies the M&A market by presuming that it is highly competitive. This may be a serviceable assumption in some instances, but in others the population of bidders may be relatively limited. In more limited settings, the ex ante bargain between common and preferred over the firm’s governance structure may affect a third-party acquirer’s bidding strategy. Here, there may be an incentive for the common and preferred to alter their governance or damages regime in a way that induces the bidder to increase its price.\textsuperscript{13} In such settings, many of our results remain intact, but the parties would have a stronger incentive to vest control over exit to the common (the constituency more reluctant to exit). Here, efficiency-minded courts may also scrutinize whether giving “too much” control to common forestalls value-adding transactions.

Third, our analysis does not consider renegotiation of the capital structure at the moment of a bid. In principle, renegotiation introduces the Coasean possibility that common and preferred shareholders reallocate their rights whenever a joint-value-increasing bid arrives. This is a potentially important consideration, and we intend to add a renegotiation stage in a later draft. However, we note that renegotiation in these situations can be cumbersome and costly, and it does not always succeed.\textsuperscript{14} Further, as repeat players, venture capital firms may have an incentive to cultivate a reputation of never renegotiating capital structure (in order to increase the credibility of future commitments). In any case, legal rules that approximate efficient outcomes can potentially save considerable transaction costs. Such rules are therefore worth pursuing even when settlement or renegotiation is possible.

Our analysis proceeds as follows. In Section 2, we develop our theoretical framework for analyzing the incentives and decisions of common and pre-

\textsuperscript{13}See Aghion and Bolton (1987); Spier and Whinston (1995).
\textsuperscript{14}Spier and Whinston (1995).
ferred shareholders in a canonical VC capital structure. There we show our baseline results related to the skewed incentives of both constituencies, as well as the conditions under which a damages regime exists to induce "efficient breach" by the party in control. Section 3 presents our analysis of ex ante contracting, demonstrating that when investment capital is scarce, privately optimal contracts may not be efficient. In Section 4 [to be written], we consider a variety of extensions to the model, including other alternatives to the pure "common chooses" or "preferred chooses" regimes. We also consider how to go about designing optimal immutable liability rules when other parameters of the contract between common and preferred (such as liquidation preference and exit control) remain subject to bargaining. Section 5 [also to be written] concludes.

2 Setup of the Model

We consider a single firm with no debt that is capitalized with common and preferred shares. All actors are assumed risk-neutral and discount time continuously at rate $r$. Our setting involves three stages of decisions, denoted $t = -2$, $t = -1$, and $t = 0$, and a terminal stage, denoted $t = T$.

At time $t = -2$, the firm must raise startup capital in the amount of $I > 0$. We assume that common shareholders (representing founders and early employees) are capital constrained. Thus, this entire amount must come from preferred investors. The preferred investors make a take-it-or-leave-it contract offer to the common at this stage. In addition to the initial investment, the terms of this offer include: (i) a liquidation preference for the preferred in the amount $K > 0$, (ii) a transfer payment of $\tau \geq 0$ from preferred to common and (iii) a rule specifying which shareholder class (common or preferred) has power to decide whether to exit/liquidate early. For now, we assume that it is prohibitively costly to renegotiate the terms of this agreement.  \(^{15}\)

At time $t = -1$, common shareholders decide whether to reject this funding (and receive zero payoff) or accept it. If they accept it, then they must also decide whether to expend noncontractable effort on behalf of the firm. We assume that the total cost of this effort is $\omega > 0$. If the common expend

\(^{15}\)This may be, for example, because the preferred shareholder is a venture capital firm that would suffer a large reputational harm for renegotiating. We consider the implications of this assumption below.
effort, then the terminal value of the firm is as described below. If they do not expend effort, then the value of the company is zero and the initial capital investment is lost.

At time $t = 0$, a potential third-party buyer emerges and the firm has an opportunity to engage in an early exit. The potential buyer first observes its willingness to pay for the firm, $v$, from a commonly-known cumulative distribution function $F(\cdot)$. The buyer then submits a take-it-or-leave-it offer, denoted $S_L$, to purchase the firm. Depending on their agreed governance structure, either common or preferred choose whether to accept or reject. If the bid is accepted, the firm is immediately liquidated at price $S_L$, preferred receive their liquidation preference $K$, and common receive the residual. If the bid is rejected, the firm continues to operate until time $t = T$, at which point the firm’s terminal value is realized.

The firm’s terminal liquidation value is the realization of a random variable $\psi_T$ drawn from a distribution with c.d.f. $G(\cdot)$, which has strictly positive support over the interval $[0, \infty)$ and is twice-differentiable. Let $S_T = E(\psi_T)$ denote the expected terminal liquidation value and $S_0 \equiv PV(S_T)|_{t=0} = e^{-rT}S_T$ the present value of the firm as a going concern as of $t = 0$. Also let $c(K|S_0)$ denote as of $t = 0$ the value of a call option on the terminal value of the firm at strike price $K$ conditional on firm’s present value being $S_0$.

Table 1 summarizes the payoffs at $t = 0$ for common shareholders, preferred shareholders, and the buyer. It assumes that the preferred makes the initial investment and that common expends effort.

<table>
<thead>
<tr>
<th>Action</th>
<th>Player</th>
<th>Continue firm</th>
<th>Liquidate firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferred</td>
<td>$S_0 - c(K; T</td>
<td>S_0)$</td>
<td>$\min{K, S_L}$</td>
</tr>
<tr>
<td>Common</td>
<td>$c(K; T</td>
<td>S_0)$</td>
<td>$\max{0, S_L - K}$</td>
</tr>
<tr>
<td>Buyer</td>
<td>0</td>
<td>$v - S_L$</td>
<td></td>
</tr>
</tbody>
</table>

$^{16}$We assume that $F(\cdot)$ exhibits standard monotone hazard rate properties
3 The Conflict Between Common and Preferred

We begin by demonstrating that, in the absence of legal liability, common and preferred shareholders will often disagree over whether the firm should liquidate or continue. As a baseline, we assume that capital markets are fully efficient and thus the potential buyer’s bid is equal to its valuation \( S_L = v \). In this case, the efficient rule is to liquidate the firm if and only if

\[
S_0 \leq S_L. \tag{1}
\]

When common shareholders control the exit decision, they favor continuation too often relative to the efficient rule. To see this, note that the common will choose to sell only if recoups more through selling than the value of its continuation option:

\[
\max\{S_L - K, 0\} \geq c(K|S_0). \tag{2}
\]

When \( S_L \leq K \), the condition above requires the value of the call option to be (weakly) negative, which cannot occur given the infinite support on \( v \). Thus, common would never exit if the offered price fell short of \( K \). Further, although common might accept an offer exceeding \( K \), they still tend favor continuation inefficiently. From the above condition, note that if \( S_L > K \), common will exit only when:

\[
S_L \geq c(K|S_0) + K \tag{3}
\]

At the same time, put-call parity requires that:

\[
c(K|S_0) + PV(K) = p(K|T|S_0) + S_0, \tag{4}
\]

where \( p(\cdot) \) denotes the value of a put option on the firm at strike price \( K \). However, since \( PV(K) < K \) and \( p(\cdot) > 0 \), it must follow that:

\[
c(K|S_0) + K > S_0, \tag{5}
\]

and thus \( S_L \) must be inframarginally above \( S_0 \) in order to induce an exit decision. In other words, common are categorically too reluctant to exit relative to the efficient rule.

\[\text{\ding{115}}\] Later we will analyze the case where capital markets are not fully efficient, so that the outside bidder may attempt to capture surplus by setting \( S_L < v \).
Figure 1: Payoffs to liquidating versus continuing the firm. The payoffs to liquidating the firm are depicted by the thick red line. The payoffs to continuing the firm are given by the thin blue line.

Now consider the case where preferred shareholders control the exit decision. Here, the opposite logic applies and preferred tend to favor exit too frequently. To see this, note that preferreds favor continuation only when the outside bid falls short of their private continuation value:

\[ S_L < S_0 - c(K|S_0), \tag{6} \]

which is strictly less than \( S_0 \) since the value of the call must be strictly positive. Intuitively, preferred categorically prefers exit (regardless of whether it is efficient) whenever \( S_L \geq K \) because liquidation gives preferred its maximum possible payoff \( (K) \). And even when \( S_L < K \), preferreds fail to internalize the costs that exit imposes on common shareholders.

Figure 1 graphically depicts the conflict. The left panel graphs common’s payoffs to liquidating given \( S_L \) (the thick dashed line) and its payoffs to continuing given \( S_0 \) (the thin solid line). For any given bid, \( S_L \), the value of \( S_0 \) that leaves common indifferent between continuing and liquidating either lies strictly below \( S_L \) (when \( S_L > K \)) or does not exist (when \( S_L \leq K \)). The right panel similarly graphs preferred’s payoffs to liquidating and continuing. For preferred, the opposite relation holds. For any given bid, \( S_L \), the value of \( S_0 \) that leaves preferred indifferent between continuing and liquidating either lies strictly above \( S_L \) (when \( S_L < K \)) or does not exist (when \( S_L \geq K \)).

14
The skewed incentives of the common and preferred shareholders demonstrated above holds true for any non-trivial value of $K \in (0, \infty)$. Only in extreme cases (i.e., where either preferred or common own the firm’s cash flows outright) does the incentive problem dissipate.

4 Optimal Contracting Under a Common Chooses Rule (“cc”)

We next back up a stage to consider the optimal contract in the absence and then presence of legal duties related to wrongful continuation and wrongful exit. Absent damages, the contractual tools in play are the chooser rule (whether common or preferred control the decision), the liquidation preference ($K$), and the up-front transfer ($\tau \geq 0$). When liability and legal duties are added to the mix, the contractual tools also include a damages rule $(D_c; D_p)$, which is described below.

This section first considers the governance regime in which common control the exit decision. We will use the abbreviation “cc” to denote this rule. The next section considers the case in which preferred control the exit decision (denoted “pc”).

4.1 No Liability

Suppose first that there is no liability for the chooser. Before the outside bid is revealed, the continuation payoff for common shareholders at $t = 0$ is:

$$
\Pi^{cc}_c (K) = F(K + c(K|S_0)) \cdot c(K|S_0) + \int_{K+c(K|S_0)}^{\infty} (v - K) dF(v) \quad (7)
$$

and the corresponding continuation value for preferred shareholders is:

$$
\Pi^{cc}_p(K) = F(K + c(K|S_0)) \cdot (S_0 - c(K|S_0)) + (1 - F(K + c(K|S_0))) K. \quad (8)
$$
Finally, the total value of the firm is the sum of these two valuations:

\[
\Pi_{cc}(K) = \Pi_{cc}^c(K) + \Pi_{cc}^p(K) = S_0 + \int_{K+c(K|S_0)}^{\infty} (v-S_0) \cdot dF(v).
\]

(9) \hspace{1cm} (10)

4.1.1 Socially Efficient Contract

Under a \textit{cc} rule with no damages, the equilibrium value of the firm is strictly decreasing in \(K\). To see this, note that:

\[
\frac{d\Pi_{firm}}{dK} = (S_0 - c(K|S_0) - K) \cdot \left(1 + \frac{dc(K|S_0)}{dK}\right) \cdot f(K+c(K|S_0)). \]

(11)

As shown above, put-call parity implies that the first term of this expression is strictly negative for all \(K > 0\). Thus, the efficient value for \(K\) (holding aside liability the parties’ incentive/participation constraints) is zero. This is functionally equivalent to selling the firm to the common shareholders. At \(K = 0\), it is easily confirmed that common will choose exit (efficiently) if and only if \(S_L \geq S_0\).

4.1.2 Privately Optimal Contract

The contract that emerges in equilibrium is not socially efficient. This point is easy to see, since the preferred would receive nothing if their liquidation preference were \(K = 0\). More formally, at time \(t = -2\), the preferreds’ contracting problem is as follows:

\[
\max_{K,\tau} \left\{ e^{-r\Pi_{cc}^p(K)} - \tau \right\} \quad \text{s.t.} \quad \begin{cases} 
(1) & \Pi_{cc}^c(K) - \omega \geq 0 \\
(2) & \tau \geq 0 \\
(3) & e^{-r} (\Pi_{cc}^c(K) - \omega) + \tau \geq 0 
\end{cases}
\]

(12)

Constraint (1) reflects the commons’ incentive constraint, requiring their expected payoff to exceed the effort costs; constraint (2) is the commons’ liquidity constraint, reflecting the fact that ex ante transfers can go only from the preferred to the common (and not in the other direction); and
constraint (3) reflects the common’s ex ante participation constraint. Note that satisfying constraint (1) is a sufficient condition for satisfying constraint (3). Thus, (3) never binds and can be ignored in what follows. In turn, this observation also suggests that $\tau = 0$, since (after elimination of constraint (3)) $\tau$ acts only to reduce the preferreds’ payoff.\(^{18}\)

We can therefore concentrate on the “relaxed” problem, eliminating the constraints to reformulate the preferred’s problem as:

$$\max_K \left\{ \Pi_p^e(K) \right\} \quad \text{s.t.} \quad \Pi_c^e(K) - \omega \geq 0$$

(13)

Analysis of this problem yields the result that the preferred’s optimal contract sets $K$ too high relative to the social optimum. To see this, consider first the preferreds’ choice of $K$ when $\omega$ is sufficiently low that the commons’ incentive constraint in (1) does not bind. Here, the optimal liquidation preference that emerges from the contract is characterized by the condition:

$$\frac{d\Pi_p^e}{dK} = \frac{d\Pi_{\text{firm}}^e}{dK} + \left[1 - F(K + c(K|S_0))\right] F(K + c(K|S_0)) \left(-\frac{dc(K|S_0)}{dK}\right)$$

(14)

$$= 0$$

Note that the second and third terms in the expression are strictly positive for all $K \geq 0$, and that $\frac{d\Pi_{\text{firm}}^e}{dK} \bigg|_{K=0} = 0$. The preferred will have an incentive to push $K$ higher than zero. Thus, at an optimum, $d\Pi_{\text{firm}}^e/dK < 0$. This inefficiency comes from the common shareholders’ liquidity constraint: If it were possible to set $\tau < 0$, then the preferred could “cash out” the efficiency gains from an optimal $K$ by extracting an up-front transfer from the common. However, because $\tau$ is constrained to be nonnegative, the preferred chooses to extract its entire surplus (inefficiently) through a highly protective $K$.

Next consider the case where $\omega$ increases to the point where the common shareholders’ incentive constraint (1) binds. The preferreds’ problem grows slightly more complex, and can be expressed as a Lagrangian (with multiplier

\(^{18}\)Note that an implementable contract also requires that $\Pi_{\text{firm}}^e \geq \omega + I$. \[17\]
\(\lambda\): 
\[
\max_{K,\lambda} \left\{ \Lambda(K, \lambda) \right\} = \max_{K,\lambda} \left\{ \Pi_p^{cc}(K) + \lambda (\Pi_c^{cc}(K) - \omega) \right\} 
\]

(15)

Here, the associated first order conditions are:
\[
\frac{d\Lambda}{dK} = \frac{d\Pi_{firm}^{cc}}{dK} + [1 - F(K + c(K|S_0))] 
+ F(K + c(K|S_0)) \left( -\frac{dc(K|S_0)}{dK} \right) (1 - \lambda) 
\]

(16)

and
\[
\frac{d\Lambda}{d\lambda} = (\Pi_c^{cc}(K) - \omega) = 0 
\]

(17)

Note that the Lagrange multiplier \(\lambda > 0\) acts to offset some of the strategic attraction to greater-than-optimal liquidation preferences. We thus predict that the optimal \(K\) is decreasing in \(\omega\). However, the privately optimal contract generally overliquidates \(K\). Thus, regardless of whether the common shareholders’ incentive constraint binds, \(K\) is overliquidated for all implementable contracts.

4.2 Liability for “Wrongful Continuation.”

Now consider an extension of the common-chooses regime to add the potential for liability. Specifically, suppose preferred could recover damages in the amount \(D_c \geq 0\) should common choose “wrongfully” to continue. The damage level, \(D_c\), may be agreed upon by the parties or provided as an implied term by a court.

In the shadow of this liability rule, the common will now choose to sell whenever:

\[
c(K + D_c|S_0) \leq \max\{0, S_L - K\}. 
\]

(18)

Note that the sole effect of damages is to increase the effective strike price (i.e., the liquidation preference of the preferred) if the firm continues. Just as before, the common will choose to sell only when:

\[
S_L \geq K + c(K + D_c|S_0). 
\]

(19)
The payoff for common shareholders under this regime as of \( t = 0 \) is:

\[
\Pi^c_{cc}(K, D_c) = F(K + c(K + D_c|S_0)) \cdot c(K + D_c|S_0)
\]

\[
+ \int_{K+c(K+D_c|S_0)}^\infty (v - K) \, dF(v)
\]

(20)

and the corresponding payoff for preferred shareholders is:

\[
\Pi^c_{cp}(K, D_c) = F(K + c(K + D_c|S_0)) \cdot (S_0 - c(K + D_c|S_0))
\]

\[
+ (1 - F(K + c(K + D_c|S_0))) \cdot K
\]

(21)

Finally, the total value of the firm is the sum of these two valuations:

\[
\Pi^c_{firm}(K, D_c) = \Pi^c_{cc}(K) + \Pi^c_{cp}(K)
\]

(22)

\[
= S_0 + \int_{K+c(K+D_c|S_0)}^\infty (v - S_0) \cdot dF(v)
\]

(23)

### 4.2.1 Socially Efficient Contract

With an additional design variable added to the mix it is now possible to consider once again the socially efficient contract as a function of \( K \) and \( D_c \). The first order conditions associated with maximizing \( \Pi^c_{firm} \) are:

\[
\frac{d \Pi^c_{firm}(K, D_c)}{dK} = \left( S_0 - c(K + D_c|S_0) - K \right) \underbrace{(+/-)}_{(+)} \cdot \left( 1 + \frac{dc}{dK} \right) \cdot f(K + c(K + D_c|S_0))
\]

(24)

and

\[
\frac{d \Pi^c_{firm}(K, D_c)}{dD_c} = \left( S_0 - c(K + D_c|S_0) - K \right) \underbrace{(+/-)}_{(-)} \cdot \left( \frac{dc}{dK} \right) \cdot f(K + c(K + D_c|S_0))
\]

(25)
As with the earlier case, one way to satisfy the above conditions and induce efficient decisions is to fix \( K = D_c = 0 \). This effectively requires self-financing by the common. However, just as before, this is not feasible given common shareholders’ liquidity constraint. With the inclusion of \( D_c \), however, this is not the exclusive way to induce efficient behavior. Indeed, so long as \( K \in [0, S_0] \), it can be shown that there is a locus of \((K, D_c)\) combinations that induce first-best efficiency, characterized by the condition:

\[
c(K + D_c | S_0) + K = S_0.
\]  

(The proof of this consists of noting that \( c(\cdot)|_{D_c=0} = S_0, c(\cdot)|_{K>0, D_c=0} > S_0 - K; c(\cdot)|_{D_c=\infty} = 0 \), and that \( c(\cdot) \) is continuous \( D_c \) for all \( K \). It then follows that for each \( K \in [0, S_0] \) there exists a unique \( D_c \) satisfying the above condition.) Let \( \delta(K) \) denote the unique value of value of \( D_c \) that satisfies this condition, so that:

\[
c(K + \delta(K) | S_0) + K = S_0
\]  

For any \( K \in [0, S_0] \), fixing \( D_c = \delta(K) \) removes the incentive for common to continue if the bid ever exceeds the going concern value of the company at \( t = 0 \). Note \( \delta(K) \) is strictly increasing in \( K \). Also note that \( \delta(K) \to \infty \) as \( K \to S_0 \), and so that the efficient damages rule rule is uniquely enforced by an injunction (or, full disgorgement of all of common’s gains from continuation). When \( K > S_0 \), however, first-best efficiency is no longer possible (though the second-best damages would still be the limiting case of \( K \to \infty \)).

### 4.2.2 Privately Optimal Contract

Will the efficient rule emerge from private contracting? To check this, note that the preferred’s (relaxed) contract design problem is:

\[
\max_{K,D_c,\tau} \{ \Pi^c_p(K, D_c) \} \quad \text{s.t.} \quad \\
(1) \quad \Pi^c_c(K, D_c) - \omega \geq 0
\]  

\[28\]
where, recall from above that:

$$\Pi_{cc} (K, D_c) = F (K + c(K + D_c|S_0)) \cdot (S_0 - c(K + D_c|S_0))$$

$$+ (1 - F (K + c(K + D_c|S_0))) \cdot K$$

(29)

$$\Pi_{cc} (K, D_c) = F (K + c(K + D_c|S_0)) \cdot c(K + D_c|S_0)$$

$$+ \int_{K+c(K+D_c|S_0)}^{\infty} (v - K) \, dF(v)$$

(30)

As noted above, constraint (1) clearly binds, this problem can be represented by a Lagrangian:

$$\max_{K,D_c,\lambda} \{ \Lambda (K, D_c, \lambda) \} = \max_{K,D_c,\lambda} \{ \Pi_{cc}^{cc} (K, D_c) + \lambda (\Pi_{cc}^{cc} (K, D_c) - \omega) \}$$

(31)

The associated first order conditions for a fully interior optimum are:

$$\frac{d\Lambda}{dK} = \frac{d\Pi_{firm}^{cc}}{dK} + F (K + c(K + D_c|S_0)) \left( -\frac{dc}{dK} \right) (1 - \lambda)$$

$$+ (1 - F (K + c(K + D_c|S_0))) = 0$$

(32)

$$\frac{d\Lambda}{dD_c} = \frac{d\Pi_{firm}^{cc}}{dD_c} + F (K + c(K + D_c|S_0)) \left( -\frac{dc}{dD_c} \right) (1 - \lambda) = 0$$

(33)

$$\frac{d\Lambda}{d\lambda} = (\Pi_{cc}^{cc} (K, D_c) - \omega) = 0$$

(34)

Note that because the call option is a function solely of the sum \((K + D_c)\), we must always have \(\frac{dc}{dK} = \frac{dc}{dD_c}\) regardless of \(K\) and \(D_c\). Imposing this on the social-efficient set of first order conditions from above (equations 24 and 24) yields:

$$\frac{d\Pi_{firm}^{cc}}{dD_c} = \frac{d\Pi_{firm}^{cc}}{dK} - (S_0 - c(K + D_c|S_0) - K) \cdot f (K + c(K + D_c|S_0))$$

(35)

We can therefore rewrite the first two first order conditions as:

$$0 = \frac{d\Lambda}{dK} = \frac{d\Pi_{firm}^{cc}}{dK} + F (K + c(K + D_c|S_0)) \left( -\frac{dc}{dK} \right) (1 - \lambda)$$

$$+ (1 - F (K + c(K + D_c|S_0)))$$

(36)
\[ 0 = \frac{d\Lambda}{dD_c} = \frac{d\Pi_{\text{firm}}^{c}}{dK} + F(K + c(K + D_c|S_0)) \left( -\frac{dc}{dK} \right) (1 - \lambda) \]
\[- (S_0 - c(K + D_c|S_0) - K) \cdot f(K + c(K + D_c|S_0)) \] (37)

At an interior maximum, and regardless of whether constraint (3) holds, the only way that both these conditions can hold is if:

\[ 1 - F(K + c(K + D_c|S_0)) = - (S_0 - c(K + D_c|S_0) - K) \times f(K + c(K + D_c|S_0)) \] (38)

which implies

\[ K - \left( \frac{1 - F(K + c(K + D_c|S_0))}{f(K + c(K + D_c|S_0))} \right) = S_0 - c(K + D_c|S_0) \] (39)

Recall that the the first-best efficient combinations of \((K, D_c)\) satisfy:

\[ K = S_0 - c(K + D_c|S_0). \] (40)

Thus, privately optimal contract differs from the first-best combination by the hazard rate \(\frac{1-F(k)}{f(k)}\). Thus, for every fixed \(K\), the optimal contract’s damages tends to exceed \(\delta(K)\). Intuitively, the preferred recognizes the common’s inefficient proclivity to continue, and uses \(D_c\) not only to reverse it, but to overcompensate in the other direction. This induces exits that are too infrequent.

Because of the overliquidation of \(D_c\) for every \(K\), it follows that the optimal contract will tend to over-deter the common from continuing relative to the efficient standard. That is, they will tend to exit even when \(S_L < S_0\). For this reason, it may be efficient for the court to impose an immutable damages rule on the preferred.

Finally, note that at the strongest damages possible, \(D_c \to \infty\), the optimal value of \(K\) converges to:

\[ K = S_0 + \frac{1 - F(K)}{f(k)} \] (41)

which is the optimal reserve price (or monopoly price) in a first- or second-price auction. In this case, the preferreds’ optimal contract sets the liquidation value \(K\) above the going concern value for the firm.
5 Optimal Contracting Under a Preferred Chooses Rule (“pc”)

Now consider a preferred chooses rule (denoted “pc”), in which the preferred shareholders decide whether to exit or continue. In this case, preferred will tend to exit inefficiently. As before, we start with contracts that contain no liability for wrongful exit, and then add liability on in a later subsection.

5.1 No Liability

Consider first the pc rule with no added liability. As noted above, preferred favor exit if and only if the amount they would receive from the bid \( \min\{K, S_L\} \) exceeds their continuation value:

\[
\min\{K, S_L\} > S_0 - c(K|S_0) \tag{42}
\]

Recall as well that when \( S_L > K \), preferred will always choose exit, and when \( S_L \leq K \), preferred choose exit only when \( S_L > S_0 - c(K|S_0) \). This gives rise to the following payoff for preferred:\footnote{Recall that put-call parity ensures that we only have to analyze the case of \( K > S_0 - c(K|S_0) \).}

\[
\Pi_{pc}^p(K) = F(S_0 - c(K|S_0))(S_0 - c(K|S_0)) + \int_{S_0 - c(K|S_0)}^{K} v dF(v) + (1 - F(K)) \cdot K \tag{43}
\]

and associated payoff for the common:

\[
\Pi_{pc}^c(K) = F(S_0 - c(K|S_0)) \cdot c(K|S_0) + \int_{K}^{\infty} (v - K) dF(v). \tag{44}
\]

The total value of the firm is:

\[
\Pi_{\text{firm}}^{pc}(K) = F(S_0 - c(K|S_0)) S_0 + \int_{S_0 - c(K|S_0)}^{\infty} v dF(v) \tag{45}
\]
5.1.1 Socially Efficient Contract

Under a preferred chooses rule, expected firm value is strictly increasing in $K$. This is because

$$\frac{d\Pi_{\text{firm}}^{pc}}{dK} = f(S_0 - c(K|S_0)) \cdot \left( -\frac{dc}{dK} \right) \cdot c(K|S_0) > 0$$ \hspace{1cm} (46)

This is the opposite of the common chooses case. (In the common choose case, the value of the firm was decreasing in $K$.) Ignoring all other constraints, the efficient allocation rule drives $K \rightarrow \infty$, and the firm is effectively sold to the preferred at the ex ante stage with no residual claim for common. However, transferring the firm to preferred would violate the incentive compatibility constraint of the common, and thus contracting will not produce the above result.

5.1.2 Privately Optimal Contract

At the ex ante stage, the preferred’s optimal contract solves the program:

$$\max_{K,\tau} \left\{ e^{-r\Pi_p^{pc}(K)} - \tau \right\} \hspace{1cm} s.t. \hspace{1cm} \begin{align*}
(1) & \quad \Pi_p^{pc}(K) - \omega \geq 0 \\
(2) & \quad \tau \geq 0 \\
(3) & \quad e^{-r}(\Pi_c^{pc}(K) - \omega) + \tau \geq 0
\end{align*} \hspace{1cm} (47)$$

Just as before, satisfying (1) and (2) implies that (3) is is not binding, and it may be ignored. Furthermore, without condition (3), condition (2) also becomes slack. So the problem becomes:

$$\max_K \left\{ \Pi_p^{pc}(K) \right\} \hspace{1cm} s.t. \hspace{1cm} (1) \quad \Pi_c^{pc}(K) - \omega \geq 0 \hspace{1cm} (48)$$

Suppose first that the incentive compatibility condition (1) is not binding. Taking derivatives with respect to $K$ yields:

$$\frac{d\Pi_p^{pc}}{dK} = F(S_0 - c(K|S_0)) \left( -\frac{dc}{dK} \right) + (1 - F(K)) > 0$$ \hspace{1cm} (49)
and

$$\frac{d\Pi_{pc}}{dK} = f (S_0 - c (K|S_0)) \cdot c (K|S_0) \cdot \left( - \frac{dc}{dK} \right)$$

$$- F (S_0 - c (K|S_0)) \cdot \left( - \frac{dc}{dK} \right) - (1 - F (K)) \cdot (50)$$

If the preferred could ignore constraints, it would choose to drive $K \to \infty$. This is clearly not feasible as it would leave the common with zero payoff. Hence, the incentive compatibility constraint (1) must be binding.

The preferred’s contracting problem can thus be summarized with a Lagrangian:

$$\max_{K, \lambda} \{ \Lambda (K, \lambda) \} = \max_{K, \lambda} \{ \Pi_{pc} (K) + \lambda (\Pi_{pc} (K) - \omega) \} \quad (51)$$

This gives first order conditions as follows:

$$\frac{d\Lambda}{dK} = (1 - \lambda) \left( F (S_0 - c (K|S_0)) \left( - \frac{dc}{dK} \right) + (1 - F (K)) \right)$$

$$\begin{aligned}
+ \lambda \frac{d\Pi_{pc}}{dK} = 0 \\
\end{aligned} \quad (52)$$

$$\frac{d\Lambda}{d\lambda} = \Pi_{pc} (K) - \omega = 0 \quad (53)$$

For any interior maximum, it must therefore be that $\lambda > 1$. This implies that there exists some finite $K$ that satisfies the above conditions. Intuitively, the preferred choose $K$ just low enough to induce effort by the common, but no lower. This is inefficient because the efficient rule is to drive $K \to \infty$.

5.2 Liability for “Wrongful Exit”

Now consider the effects of a damages payment $D_p$ associated with “wrongful exit” by the preferred. The effect of this liability exposure will be to induce exit only when:

$$\min \{ K, S_L \} - D_p > S_0 - c (K|S_0) \quad (54)$$
Consider first the case where \( S_L \geq K \), so that exiting gives the preferred \( K \) at most. Here, the preferred will exit if and only if the immediate liquidation preference (net of damages) exceeds its continuation value:

\[
K - D_p > S_0 - c(K|S_0)
\]  

(55)

For sufficiently small levels of \( D_p \), this value must be positive (an artifact of put-call-parity, as shown above). But it is also possible that \( D_p \) could be so high that the preferred would be deterred from exit even if \( S_L \geq K \). However, if that were to occur, then it would also imply that the preferred would refuse to exit for any value of \( S_L < K \). That is, once \( D_p \) grows so large as to violate the above inequality, preferred never want to liquidate, which cannot be optimal. This implies that (a) for any optimal \( D_p \) must satisfy \( D_p < K \) and (b) we can restrict our attention to levels of \( D_p \) that are sufficiently “small,” that is, levels such that preferred choose to exit when \( S_L \geq K \). Given these two implications, the marginal case (in which preferred switch from liquidating to continuing) occurs when \( S_L < K \). Thus, preferred will exit if and only if:

\[
S_L - D_p > S_0 - c(K|S_0)
\]  

(56)

The payoff for preferred shareholders under this regime (measured as of \( t = 0 \)) is:

\[
\Pi_{pc}^c (K, D) = F (S_0 + D_p - c(K|S_0)) \cdot (S_0 - c(K|S_0))
\]

\[
+ \int_{S_0 + D_p - c(K|S_0)}^{K + D_p} (v - D_p) \ dF(v) + (1 - F (K + D_p)) \cdot K
\]  

(57)

and the derivates with respect to \( K \) and \( D_p \) are

\[
\frac{d \Pi_{pc}^c}{dK} = F (S_0 + D_p - c(K|S_0)) \cdot \left(-\frac{dc}{dK}\right) + (1 - F (K + D_p))
\]

(58)

\[
\frac{d \Pi_{pc}^c}{dD_p} = -[F (K + D_p) - F (S_0 + D_p - c(K|S_0))]
\]

(59)

In words, if the bid is below \( S_0 + D_p - c(K|S_0) \), then preferred continues gets its reservation value. If the bid is above \( S_0 + D_p - c(K|S_0) \), then preferred exits but it must first pay \( D_p \) to common (as judgment creditors) before it can start receiving its liquidation preference, \( K \). That leaves preferred with max \( \{S_L - D_p, K\} \).
The corresponding quantities for common are:

\[
\Pi^c (K, D) = F (S_0 + D_p - c (K|S_0)) \cdot c (K|S_0) \\
+ (F (K + D_p) - F (S_0 + D_p - c (K|S_0))) \cdot D_p \\
+ \int_{K+D_p}^{\infty} (v - K) dF (v) \quad (60)
\]

\[
\frac{d\Pi^c}{dK} = f (S_0 + D_p - c (K|S_0)) \cdot (c (K|S_0) - D_p) \cdot \left( -\frac{dc}{dK} \right) \\
- F (S_0 + D_p - c (K|S_0)) \cdot \left( -\frac{dc}{dK} \right) - (1 - F (K + D_p)) \quad (61)
\]

\[
\frac{d\Pi^c}{dD_p} = f (S_0 + D_p - c (K|S_0)) \cdot (c (K|S_0) - D_p) \\
+ F (K + D_p) - F (S_0 + D_p - c (K|S_0)) \quad (62)
\]

The total expected value of the firm at time \( t = 0 \) is:

\[
\Pi^{pc}_{\text{firm}}(K, D_p) = S_0 + \int_{S_0+D_p-c(K|S_0)}^{\infty} (v - S_0) f(v) dv. \quad (63)
\]

### 5.2.1 Socially Efficient Contract

Consider first the benchmark of first-best with \( K \) and \( D_p \). Maximizing with respect to \( K \) and \( D_p \) yields the following first order conditions:

\[
\frac{d\Pi^{pc}_{\text{firm}}}{dK} = 0 = - (D_p - c (K|S_0)) \cdot f (S_0 + D_p - c (K|S_0)) \left( -\frac{dc}{dK} \right) \quad (64)
\]

\[
\frac{d\Pi^{pc}_{\text{firm}}}{dD_p} = 0 = - (D_p - c (K|S_0)) \cdot f (S_0 + D_p - c (K|S_0)) \quad (65)
\]

Note that both of these are satisfied for any value of \( K \) if:

\[
D_p = c (K|S_0) \quad (66)
\]

Thus, as with the common choose rule, there a schedule of liquidation preferences and damages combinations that implement efficient exit decisions by preferred. The optimal damages is a decreasing function of \( K \). This goes in the opposite direction from the common-choose case. Unlike the optimal rule under the common choose rule, the optimal damages under the preferred-choose rule are equal to expectation damages.
5.2.2 Privately Optimal Contract

The preferred will solve the following problem:

\[
\max_{K, \tau} \left\{ e^{-r \Pi_{pc}^c (K, D_p)} - \tau \right\} \quad \text{s.t.} \\
(1) \quad \Pi_{pc}^c (K, D_p) - \omega \geq 0 \\
(2) \quad \tau \geq 0 \\
(3) \quad e^{-r} (\Pi_{pc}^c (K, D_p) - \omega) + \tau \geq 0 
\]

As before, this is equivalent to the relaxed problem:

\[
\max_{K, \tau} \left\{ \Pi_{pc}^c (K, D_p) \right\} \quad \text{s.t.} \\
(1) \quad \Pi_{pc}^c (K, D_p) - \omega \geq 0
\]

Since constraint (1) is binding, this problem can be restated as a Lagrangian: \(^{21}\)

\[
\Lambda(K, D_p, \lambda) = \Pi_{pc}^c (K, D_p) + \lambda (\Pi_{pc}^c (K, D_p) - \omega)
\]

It is worth noting the various derivatives of the parties in \(K\) and \(D_p\)

\[
\frac{d \Pi_{pc}^c}{d K} = F(S_0 + D_p - c(K|S_0)) \cdot \left( -\frac{dc}{d K} \right) + (1 - F(K + D_p)) \\
\frac{d \Pi_{pc}^c}{d D_d} = -[F(K + D_p) - F(S_0 + D_p - c(K|S_0))] \\
\frac{d \Pi_{pc}^c}{d K} = -\frac{d \Pi_{pc}^c}{d K} + f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p) \cdot \left( -\frac{dc}{d K} \right) \\
\frac{d \Pi_{pc}^c}{d D_p} = -\frac{d \Pi_{pc}^c}{d D_d} + f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p)
\]

The first order conditions are

\[
\frac{d \Lambda}{d K} = (1 - \lambda) \frac{d \Pi_{pc}^c}{d K} \\
+ \lambda \cdot \left( f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p) \cdot \left( -\frac{dc}{d K} \right) \right) \\
\frac{d \Lambda}{d D_p} = (1 - \lambda) \frac{d \Pi_{pc}^c}{d D_d} + \lambda \cdot (f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p))
\]

\(^{21}\)See above.
\[
\frac{d\Lambda}{d\lambda} = (\Pi_c^p(K, D_p) - \omega) = 0 \quad (76)
\]

The first two first order conditions can be rearranged as:

\[
\frac{d\Pi_c^p}{dK} = -\frac{\lambda}{1-\lambda} \cdot (f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p))
\times \left( -\frac{dc}{dK} \right) \quad (77)
\]

\[
\frac{d\Pi_c^p}{dD_d} = -\frac{\lambda}{1-\lambda} \cdot (f(S_0 + D_p - c(K|S_0)) \cdot (c(K|S_0) - D_p)) \quad (78)
\]

Dividing these expressions allows us to eliminate \(\lambda\):

\[
\left( \frac{d\Pi_c^p}{dK} \right) / \left( \frac{d\Pi_c^p}{dD_d} \right) = \left( -\frac{dc}{dK} \right) \quad (79)
\]

This implies that the privately optimal contract is a corner solution. To see this note that if there were an internal optimum, then the following relationship must hold between the optimal levels of \(K\) and \(D_p\):

\[
\frac{d\Pi_c^p}{dK} = \frac{d\Pi_c^p}{dD_d} \left( -\frac{dc}{dK} \right) \quad (80)
\]

Substituting and rearranging, we have

\[
F(S_0 + D_p - c(K|S_0)) \cdot \left( -\frac{dc}{dK} \right) + (1 - F(K + D_p)) =
\]

\[-F(K + D_p) \cdot \left( -\frac{dc}{dK} \right) + F(S_0 + D_p - c(K|S_0)) \cdot \left( -\frac{dc}{dK} \right) \quad (81)\]

which gives

\[(1 - F(K + D_p)) = -F(K + D_p) \cdot \left( -\frac{dc}{dK} \right). \quad (82)\]

However, this last expression does not hold for any finite \(K\) and \(D_p\). This is because the left hand side is always positive, while the right hand side is always negative.
There are thus two possibilities. The first and most natural is that the preferreds’ optimal contract sets $D_p = 0$. This case ($pc$ with no liability) was analyzed in the previous section. In this case, the optimal contract sets $K$ equal to the highest point that satisfies the commons’ participation constraint. As explained above, this is also socially inefficient.

The second and perhaps less natural possibility is that preferred sets $K$ arbitrarily high and then chooses $D_p$ such that commons’ participation constraint is just satisfied. Under this arrangement, the payoffs to preferred and common become

$$\Pi^p_c = -D (1 - F(S_0 + D)) + F(S_0 + D)S_0 + \int_{S_0 + D}^{\infty} vdF(v) \quad (83)$$

$$\Pi^c_c = D (1 - F(S_0 + D)) \quad (84)$$

From a cash flow perspective, this contract essentially converts preferred shareholders into common shareholders, and common shareholders into a debt claimant. Further, common’s debt claim, $D_p$, pays out only when the firm is sold early. Otherwise, common receives nothing. In this case, $K \to \infty$ and $D_p$ is the unique level that satisfies

$$D_p = \frac{\omega}{1 - F(S_0 + D_p)} \quad (85)$$

The difference in the equilibrium payoffs to preferred in the second versus the first corner solution case is

$$\Pi^*_p|_{K \to \infty} - \Pi^*_p|_{D_p = 0} = \int_{S_0 - c(K^*|S_0)}^{s_0 + D^*_p} (v - S_0)dF(v), \quad (86)$$

where $D^*_p$ satisfies equation 85 and $K^*$ is the solution to preferreds’ optimal contracting problem under the preferred choose rule and no liability (analyzed above).

6 Extensions

To be written.
7 Conclusion

To be written.

References


