Multi-Sender Disclosure of Verifiable Information[*]

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Abstract

Does competition promote disclosure? We study a class of disclosure games with multiple senders. Senders have linear preferences over a receiver's posterior expectation of some state of the world, and conditional on the true state, each sender may draw an independent verifiable signal. We show that senders' disclosure behavior are *strategic complements* when there is a cost to concealing information but *strategic substitutes* when there is a cost to disclosing information. Thus, when concealing information is costly, additional senders can only help a receiver; by contrast, when disclosure is costly, a receiver can be better off with fewer senders, less informed senders, or when senders face higher disclosure costs. These insights are derived from the following general result: when two Bayesian individuals have mutually-known different priors over the state, under suitable conditions, each believes that a more informative experiment (Blackwell, 1953) will, on expectation, bring the other's posterior expectation closer to his own prior expectation.

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1   Introduction

Decision makers often seek information from more than one interested agent. For example, judges and arbiters rely on evidence provided by both defendants and plaintiffs, CEOs are advised by various subordinates whose interests may conflict, consumers receive product information from biased sources, and legislative bills can be shaped by the information revealed by different interest groups. In all these cases, agents will strategically disclose or conceal their information in order to influence the decision maker. How are an agent's incentives and behavior affected by the presence of other agents? What are the implications for the decision maker's welfare? In particular, does a decision maker always benefit from having access to multiple sources of information? These are questions of substantial importance in a host of economic settings.

This paper aims to provide some new insights into these long-standing issues. In a class of environments, we identify a novel factor—whether disclosure or concealment of information is costly—that determines when an agent will react to other agents' information provision by disclosing more of his own information and when he will react by disclosing less. We also show that, contrary to standard intuitions, increased competition in information provision can harm even a fully-rational decision maker (hereafter, DM).

We study a family of voluntary *disclosure games* in which biased agents' only instrument of influence is the certifiable or verifiable private information they are exogenously endowed with. Agents cannot lie about their information but simultaneously choose what to disclose and what to withhold.[1] The model we develop has four salient ingredients. First, each agent has monotonic and state-independent preferences: his payoff is linear (increasing or decreasing) in the DM's expectation of an unknown "state of the world". Second, each agent is only informed with some probability, and while he can certify his information when informed (or choose to withhold it), he cannot certify that he is uninformed. This partial certifiability generally precludes full disclosure in equilibrium (Dye, 1985; Shin, 1994a,b), in contrast to the classic "unravelling" arguments of Grossman (1981) and Milgrom (1981). Third, we posit that informed agents have imperfectly correlated information; specifically, they draw signals that are independently distributed conditional on the state. While this is a prevalent assumption in numerous areas of information economics, much of the existing work on disclosure games assumes that informed agents have identical information (e.g. Milgrom and Roberts, 1986; Lipman and Seppi, 1995; Shin, 1998; Bourjade and Jullien, 2011; Bhattacharya and Mukherjee, 2013).[2]

Fourth, and most importantly, we allow for *message costs*: for each agent, either disclosure or con-

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[1]Such games are also referred to as "persuasion games"; see Milgrom (2008) and Dranove and Jin (2010) for surveys. A parallel literature initiated by Crawford and Sobel (1982) and Green and Stokey (2007) studies "cheap-talk" games where agents can send arbitrary messages.

[2]Two exceptions are Okuno-Fujiwara et al. (1990) and Hagenbach et al. (2013); these papers identify conditions for full disclosure and hence focus on different issues than we do. Another exception is Jackson and Tan (2013), which we comment on in more detail later. There are also a number of papers that look at the effects of competition on quality disclosure in markets where firms compete in prices; even though some of this work allows firms to hold heterogenous information, price competition introduces considerations that are quite different from pure disclosure games.

cealment of information can entail direct costs, independent of the influence on the DM's belief. Disclosure costs are natural when the process of certifying or publicizing information demands resources such as time, effort, or hiring an outside expert. A subset of the literature, starting with Jovanovic (1982) and Verrecchia (1983), has modeled such a cost, although primarily only in single-agent problems. On the flip side, there are contexts in which it is the suppression of information that requires costly resources; alternatively, there may be a psychic disutility to concealing known information, or concealment may be discovered ex-post (by auditors, whistleblowers, mere happenstance) and result in negative consequences for the agent through explicit punishment or reputation loss.[3] Somewhat surprisingly, there has not been any systematic treatment of concealment costs in the existing disclosure literature (to our knowledge); note, though, that potent mandatory disclosure laws can be viewed as inducing large concealment costs.[4]

The central theme of our findings is that the strategic interplay between agents in their disclosure of information turns critically on whether there are message costs, and if so, of which kind. Consequently, the nature of message costs has significant implications for the DM's welfare when there are changes in the environment such as an increase in the number of agents, an increase in the message cost, or agents become better informed.

To explain our results, it is useful to begin with a benchmark irrelevance result when there are no message costs. Consider the disclosure game with just one agent, say i. Regardless of disclosure/concealment costs, agent i's equilibrium disclosure behavior follows a simple threshold rule: he discloses all signals that are sufficiently favorable to his cause and conceals the complementary set of sufficiently unfavorable signals. Without message costs, the threshold signal, say s_i^*, is such that the DM's belief in the absence of disclosure—her *nondisclosure belief*—must be equal to the belief induced by disclosure of the threshold signal.[5] Agent i is able to avoid complete skepticism from the DM when she does not disclose information because the DM has to assign some probability to i being genuinely uninformed.

Now add the presence of another agent, say j. We show that without message costs, it would remain optimal for i to continue using exactly the same threshold no matter how j behaves. The intuition is that it is as though i's disclosure choice only affects the belief that the DM uses as a "prior" for Bayesian updating from j's message. From this perspective, i's objective is simply to induce the

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[3]For example, it is now widely acknowledged that firms in the tobacco industry paid consultants, journalists, and pro-tobacco organizations to debate the evidence on the adverse effects of tobacco, and they engaged in elaborate schemes to destroy or conceal documents that contained damaging information. Ironically, irrefutable evidence of decades of these practices surfaced in the once-secret "tobacco documents" obtained in the 1990s through litigation in the United States.

[4]The idea of concealment costs bears some similarity to the falsification or lying costs that have been incorporated in various other contexts, for example by Lacker and Weinberg (1989) in a contracting problem, Kartik et al. (2007) and Kartik (2009) in strategic information transmission, or Emons and Fluet (2009) in an arbitration framework. In a credit policy context, Rajan (1994) studies a model where reputation-motivated banks can conceal their knowledge of bad loans through liberal credit policies that entail real costs.

[5]This is because signals are continuously distributed. Also, to be more precise, the object of interest is the DM's expectation of the state. Unless specified otherwise, assume for this introduction that the state is binary, so that there is a one-to-one relationship between distributions over the state and their expectation.

most favorable belief in the DM, which is the same as his objective in the single-agent problem.

How do message costs alter this irrelevance result? Agent i must now consider not only the belief he induces in the DM but also the corresponding direct cost. Consequently, in the single-agent setting, it is no longer true that the threshold type of i induces the same equilibrium belief in the DM when he discloses as when he conceals. To the contrary, if the two beliefs were the same, then the threshold type would strictly prefer to either conceal (if disclosure is costly) or disclose (if concealment is costly). Thus, i's threshold will be such that the DM's nondisclosure belief is either less favorable than the belief induced by disclosure (if disclosure is costly) or more favorable (if concealment is costly). In either case, in equilibrium, there is a *disagreement* between the DM's nondisclosure belief and i's threshold private belief; moreover, the direction of disagreement turns on whether disclosure or concealment is costly.

When agent j is now added to the picture, one can view j's message as providing the DM with a second "experiment" in the sense of Blackwell (1951, 1953); of course, due to j's strategic behavior, the precise nature of this experiment must be determined in equilibrium and will generally depend on i's own equilibrium disclosure strategy. Nevertheless, we are able to circumvent this issue by proving the following: *when two Bayesian individuals have (mutually-known) different prior expectations of the state, each believes that a Blackwell more-informative experiment will, on expectation, bring the other's posterior closer to his own prior.*[6] As this result forms the backbone of our main insights, we refer to it for brevity as *more information is expected to further validate one's prior*, or even more succinctly as just *information validates the prior* (hereafter, IVP).

Applying IVP to our strategic disclosure problem, one sees that regardless of exactly how agent j behaves, the threshold type of i expects j's presence to "close the disagreement gap", so to speak, between i's own belief and the belief he induces in the DM through nondisclosure. Due to the direction of disagreement discussed above, i's expected payoff from concealing his single-agent threshold signal is now lower when there is a concealment cost but higher when there is a disclosure cost. As in either case the expected payoff from disclosure does not change, the addition of agent j drives agent i to more disclosure when there is a concealment cost and less disclosure when there is a disclosure cost.[7] IVP further implies that these effects are stronger when agent j's discloses more (in the sense of Blackwell). Since a symmetric argument applies to agent j, it follows that the two agents' disclosure behavior are *strategic complements* when concealing information is costly, whereas they are *strategic substitutes* when disclosing information is costly.

It is worth emphasizing that these results are driven entirely by two themes: (i) message costs create

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[6]More precisely: if an individual m knows individual n's prior, then m believes that, on expectation, a Blackwell more-informative experiment will bring n's posterior closer to m's prior; see Theorem 1. When there are more than two states, the last part of the sentence would read "will bring n's posterior *expectation* closer to m's prior *expectation*", and the result requires suitable likelihood-ratio ordering assumptions; see Theorem 2. Note that it is important that the two individuals agree on what the experiments are, in a sense we will make clear. When applied to our strategic disclosure game, this is guaranteed by equilibrium analysis.

[7]In a different model, Bourjade and Jullien (2011) find an effect related to that we find under concealment cost. Loosely speaking, "reputation loss" in their model plays a similar role to concealment cost in ours.

equilibrium disagreement upon nondisclosure between an agent and the DM in the sense explained above; and (ii) whenever more information is disclosed by others, this disagreement and IVP combine to imply a systematic shift in an agent's own disclosure behavior. The direction of this shift depends only on the direction of disagreement, which in turn is determined by the nature of message costs. For this reason, it does not matter which direction any particular agent would like to push the DM's posterior, upward or downward. Furthermore, the underlying logic is *not* that of reduced "pivotality" or "free-riding" when there are additional agents; rather, it stems from the asymmetry that, under message costs, nondisclosure creates an equilibrium disagreement whereas disclosure does not.

Our results about strategic disclosure behavior have straightforward welfare implications. When concealing information is costly, a DM in our setting always benefits from having access to an additional agent not only because of the information this agent provides—the direct or marginal effect—but also because this improves disclosure from other agents—an indirect or inframarginal effect. The strategic complementarity in behavior also implies indirect benefits to a DM when any agent's concealment cost decreases or the probability that he is informed increases.

Welfare properties are quite different when agents bear costs of disclosing information. In this case, the strategic substitutes result implies that while a DM gains some direct benefit from consulting an additional agent, the indirect effect through other agents' behavior is deleterious to the DM. In general, the net effect is ambiguous. We show by example that even a DM with familiar quadratic-loss preferences can be *strictly worse off when faced with two agents rather than one*, or when some agent's disclosure cost decreases or the probability that he is informed increases—the latter two being changes that would benefit any DM when faced with only one agent. The welfare loss for the DM can arise even if the two agents are biased in opposite directions and hence, because of their linear preferences, are engaged in pure competition (i.e. a zero-sum game).

Thus, competition between agents need not increase information revelation and/or benefit rational DMs; the mechanism of strategic substitution in disclosure is, to our knowledge, new.[8] This "perverse welfare" result may be relevant in various applications for thinking about the effects of changes in the environment that appear to be improvements at first blush. For example, since disclosure costs are important in arbitration, litigation, or related judicial settings (e.g. Sobel, 1989), it qualifies arguments made in favor of adversarial procedures that are based largely on promoting disclosure of information (e.g. Shin, 1998).

Before turning to the formal analysis, we would also like to mention that while this paper applies IVP to study the interplay of strategic disclosure between multiple agents, we hope that IVP will also be useful in other contexts. Indeed, the logic of IVP underlies the mechanisms in a few existing papers

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[8]In quite different settings, Milgrom and Roberts (1986), Dewatripont and Tirole (1999), Krishna and Morgan (2001), and Gentzkow and Kamenica (2012a) offer formal analyses supporting the viewpoint that competition between agents helps—or at least cannot hurt—a DM. Elliott et al. (2012) show how a DM can be harmed by "information improvements" in a cheap-talk setting, but the essence of their mechanism is not the strategic interaction between agents. We also note that various authors study how endogenous costly information acquisition can lead to free-riding problems when there are more agents. While we assume exogenous information in this paper, see Section 4 for a comment on endogenous information acquisition.

that study models with heterogeneous priors under specific information structures.[9]

The remainder of this paper is organized as follows: Section 2 lays out the model, Section 3 develops the main results, Section 4 discusses extensions and limitations, and Section 5 concludes. Formal proofs of all results are in Appendix A, while Appendix B contains some supplementary material.

2   The Model

Players.   There is a unknown state of the world, $\omega \in \Omega \subseteq \mathbb{R}$, where Ω is a finite set. A decision maker (DM) will form a belief $\beta_{DM} \in \Delta(\Omega)$, where $\Delta(\Omega)$ is the set of probability distributions on Ω. For much of our analysis, all that matters is the belief that the DM holds; for welfare evaluation, however, it is useful to view the DM as taking an action a with von-Neumann Morgenstern utility function $u_{DM}(a, \omega)$. There is a finite set of N agents indexed by i. Each agent i has state-independent preferences over the DM's belief that are parameterized by a variable $b_i \in \{-1, 1\}$. Agent i's von Neumann-Morgenstern preferences are represented by the function $u(\beta_{DM}, b_i) = b_i \mathbb{E}_{\beta_{DM}}[\omega]$, where $\mathbb{E}_\beta[\omega]$ is the expectation of ω under distribution β. Thus, each agent has linear preferences over the DM's expectation of the state; $b_i = 1$ means that agent i is biased upward (i.e. prefers higher expectations), and conversely for $b_i = -1$. All agents' biases are common knowledge.

Information.   The DM relies on the agents for information to form her belief about the state. All players share a common prior over the state, $\pi \in \Delta(\Omega)$. Each agent may exogenously obtain some private information about the state. Specifically, with independent probability $p_i \in (0, 1)$, an agent i is informed and receives a signal $s_i \in S$; with probability $1 - p_i$, he is uninformed, in which case we denote $s_i = \phi$. If informed, agent i's signal is drawn independently from a distribution that depends upon the true state. For technical convenience, we assume that the state-contingent cumulative distributions of signals, $F(s|\omega)$ for $\omega \in \Omega$, have common support $S = [\underline{s}, \overline{s}] \subseteq [0, 1]$ and admit respective densities $f(s|\omega)$.[10]

Communication.   Signals are "hard evidence"; an agent with signal $s_i \in S \cup \{\phi\}$ can send a message $m_i \in \{s_i, \phi\}$. In other words, an uninformed agent only has one message available, ϕ, while an informed agent can either report his true signal or feign ignorance by sending the message ϕ.[11] We refer

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[9]In particular, see the strategic "persuasion motive" that generates bargaining delays in Yildiz (2004), motivational effects of difference of opinion in Che and Kartik (2009) and Van den Steen (2010, Proposition 5), and a rationale for deference in Hirsch (2013, Proposition 8). In a non-strategic setting, see why minorities expect lower levels of intermediate bias in Sethi and Yildiz (2012, Proposition 5).

[10]It is straightforward to allow for heterogeneity across agents in the state-contingent distributions from which they independently draw signals when informed; we assume homogeneity here to reduce notation as we already allow for heterogeneity in the probabilities of being informed.

[11]Given that each agent's preferences are monotonic in the DM's belief and the direction of bias is known, standard "skeptical posture" arguments imply that our results would not be affected if we were to allow for richer forms of withholding,

to any message $m_i \neq \phi$ as *disclosure* and the message $m_i = \phi$ as *nondisclosure*; furthermore, when an informed agent chooses nondisclosure, we say that he is *withholding* or *concealing* his information. The constraint that agents must either tell the truth or conceal their information is standard; a justification is that signals are verifiable and sufficiently large penalties will be imposed on an agent if a reported signal is discovered to be untrue.

Aside from affecting the DM's belief, disclosure or concealment of information may entail direct costs. Specifically, an agent i who sends message $m_i \neq \phi$ bears a utility cost $c \in \mathbb{R}$. We refer to the case of $c > 0$ as one of *disclosure cost* and $c < 0$ as one of *concealment cost*. A disclosure cost captures the idea that costly resources may be needed to certify or make verifiable the information that one has; a concealment cost captures either a resource-related or psychic disutility from concealing available information, or it could represent expectations of possible penalties from ex-post detection of having withheld information. As is well known, the presence of a disclosure cost precludes full disclosure in settings like this one. For this reason, our main points under $c > 0$ do not require the assumption that each agent may be uninformed. We make that assumption to allow for a unified treatment of both $c > 0$ and $c \leq 0$.[12]

Contracts and Timing. As is standard in the incomplete contracts literature, we rule out any transfers that are contingent on messages or decisions; implicitly, this presumes that even though signals are verifiable by the DM, they are not verifiable by a third party or, for other reasons, are non-contractible. We also assume that no player has any commitment power. The game we study is therefore the following: nature initially determines the state ω and then independently (conditional on the realized state) draws each agent i's private information, $s_i \in S \cup \{\phi\}$; all agents then simultaneously send their respective messages m_i to the DM (whether messages are public or privately observed by the DM is irrelevant); the DM then forms her belief, β_{DM}, according to Bayes rule, whereafter each agent i's payoff is realized as

$$u(\mathbb{E}_{\beta_{DM}}[\omega], b_i) - c \cdot \mathbb{1}_{\{m_i \neq \phi\}}.$$

All aspects of the game except the state and agents' signals (or lack thereof) are common knowledge. The solution concept we adopt is Perfect Bayesian Equilibrium (Fudenberg and Tirole, 1991), which we will refer to simply as "equilibrium". The notion of welfare we use for any player is ex-ante expected utility.

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for example if an informed agent could report any subset of the signal space that contains his true signal. Likewise, also allowing for cheap talk would not affect our results as cheap talk would be uninformative in equilibrium.

[12]When $c = 0$ our setting is related to Jackson and Tan (2013) in the following way: their multiple expert/single voter setting is like ours, except they assume binary signals and binary decisions. Because the decision space is binary, their experts' payoffs are not linear in the voter's posterior, which substantially alters the thrust of the analysis. The substantive focus is also orthogonal: they are ultimately interested in comparing different voting rules, which is effectively like changing the (pivotal) voter's preferences; we, on the other hand, highlight how the presence of disclosure or concealment costs affects the strategic interaction between agents holding fixed the DM's preferences. When $c = 0$, our setting is also related to Bhattacharya and Mukherjee (2013). They assume that informed agents' signals are perfectly correlated but allow for agents to have non-monotonic preferences over the DM's posterior.

3 Analysis

To convey our main points most transparently, we assume in this section that the state space is binary, $\Omega = \{0, 1\}$. Accordingly, we identify all beliefs over the state with the probability they place on state 1, which is also their expectation. Similarly, we assume that there are only two agents, $N = 2$. Section 4 discusses the generalizations to any finite number of states and agents.

In the binary-state setting, it is convenient and and without much loss of generality to equate signals with private beliefs, i.e. to assume that an agent's posterior on the state $\omega = 1$ given only his own signal s_i (as derived by Bayesian updating from the prior and the signal distributions) is s_i.[13]

3.1 A single-agent game

Begin by considering a hypothetical game between a single agent i and the DM; for concreteness, suppose this agent is upward biased (straightforward analogs of the discussion below apply if the agent is downward biased, as we will mention). For any $\beta \in [0, 1]$, define $f_\beta(s) := \beta f(s|1) + (1 - \beta)f(s|0)$ as the unconditional density of signal s given a belief that puts probability β on state $\omega = 1$. Let F_β be the cumulative distribution of f_β.

Trivially, disclosure of signal s will lead to the DM holding belief s. It follows that given any *nondisclosure belief*, i.e. the DM's posterior belief when there is nondisclosure, the optimal strategy for the agent (if informed) is a threshold strategy of disclosing all signals above some *disclosure threshold*, say $\hat{s}$, and concealing all signals below $\hat{s}$;[14] if the agent is uninformed, he only has one message available. Conversely, suppose the agent is using a disclosure threshold $\hat{s}$. Define the function $\eta : [0, 1] \times (0, 1) \times [0, 1] \to [0, 1]$ by

$$\eta(\hat{s}, p, \pi) := \frac{1 - p}{1 - p + pF_\pi(\hat{s})}\pi + \frac{pF_\pi(\hat{s})}{1 - p + pF_\pi(\hat{s})}\mathbb{E}_\pi[s|s < \hat{s}], \tag{1}$$

where $\mathbb{E}_\pi[\cdot]$ refers to an expectation taken with respect to the distribution F_π. This function is simply a posterior derived from Bayes rule in the event of nondisclosure using the prior π, a probability p of the agent being informed, and a conjectured disclosure threshold $\hat{s}$.

An increase in the agent's disclosure threshold has two effects on the DM's nondisclosure belief: first, it increases the likelihood that nondisclosure is due to the agent concealing his signal rather than being uninformed; second, conditional on the agent in fact concealing his signal, it causes the DM

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[13]The only loss of generality is that we are now ruling out atoms on any private belief due to the maintained assumption that the distribution of signals in each state admits a density. We could work directly with the underlying signal structure instead and allow for distinct signals to induce the same private belief, which would permit us to subsume discrete signals as in Pesendorfer and Swinkels (1997, p. 1253). We choose not to do so for our general analysis because this would require more elaborate statements at certain points (such as Lemma 2) without adding commensurate insight.

[14]This corresponds to what Shin (1994a,b) calls a "sanitization strategy". Note that the agent's behavior at the threshold signal is immaterial because the distribution of signals is atomless.

to expect a higher signal, i.e., to raise her belief. As the DM's belief conditional on concealment is lower than the prior (since the agent is using a threshold strategy), these two effects work in opposite directions. On the other hand, holding the disclosure threshold fixed, an increase in the probability that the agent is informed has an unambiguous effect because it increases the probability that nondisclosure is due to concealed information rather than no information.

Lemma 1. *The nondisclosure belief function, $\eta(\hat{s}, p, \pi)$, has the following properties:*

1. *It is single-troughed in $\hat{s}$, i.e. it is strictly decreasing when $\hat{s} < \eta(\hat{s}, p, \pi)$ and strictly increasing when $\hat{s} > \eta(\hat{s}, p, \pi)$. Consequently, there is a unique solution to $\hat{s} = \eta(\hat{s}, p, \pi)$, and this solution is interior.*

2. *It is weakly decreasing in p, strictly if $\hat{s} \in (\underline{s}, \overline{s})$.*

(All proofs are in Appendix A.)

It follows from the above discussion that any equilibrium is fully characterized by the disclosure threshold the agent uses in the equilibrium. If this threshold is interior, the agent must be indifferent between disclosing the threshold signal and concealing it. As the agent's payoff from disclosing any signal s is $s - c$ (recall that c is positive when there is a disclosure cost and negative when there is a concealment cost), we obtain the following equilibrium characterization.

Proposition 1. *Assume there is only one agent i, and this agent is biased upward.*

1. *Any equilibrium has a disclosure threshold s^* such that: (i) s^* is interior and $\eta(s^*, p_i, \pi) = s^* - c$; or (ii) $s^* = \underline{s}$ and $\pi \leq \underline{s} - c$; or (iii) $s^* = \overline{s}$ and $\pi \geq \overline{s} - c$. Conversely, for any s^* satisfying (i), (ii), or (iii), there is an equilibrium with disclosure threshold s^*.*

2. *If there is either no message cost or there is a concealment cost ($c \leq 0$) then there is a unique equilibrium. Furthermore, if there is no message cost ($c = 0$) the equilibrium disclosure threshold is interior.*

3. *If there is a disclosure cost ($c > 0$) then there can be multiple equilibria.*

Part 1 of the proposition is straightforward; Parts 2 and 3 build on Lemma 1. Multiple equilibria can arise under a disclosure cost because, in the relevant domain (to the right of the fixed point of $\eta(\cdot, p_i, \pi)$), the DM's nondisclosure belief is increasing in the agent's disclosure threshold. In such cases, we will focus on properties of the highest and lowest equilibria in terms of the disclosure threshold. Intuitively, these equilibria respectively correspond to when the agent is least and most informative, and are thus respectively the worst and best equilibria in terms of the DM's welfare; we defer a formal treatment of these points to Subsection 3.3. On the other hand, the ranking of those equilibria is reversed for the agent's welfare. To see this, note that because the agent's preferences are linear in the DM's belief and we evaluate welfare at the ex-ante stage, the agent's welfare in an equilibrium with threshold s^* is $\pi - p_i(1 - F(s^*))c$. Thus, when $c > 0$, the agent's welfare is higher when the disclosure threshold is

higher: at the ex-ante stage, he cannot affect the DM's belief on expectation and thus would prefer to minimize the probability of incurring the disclosure cost.

When $c = 0$, the agent's belief when he receives the threshold signal is identical to the DM's equilibrium nondisclosure belief. On the other hand, when $c \neq 0$ these two beliefs will differ in any equilibrium: if $c > 0$, the agent's threshold belief, s^*, is higher than the DM's nondisclosure belief, $\eta(s^*, p_i, \pi)$, and conversely when $c < 0$. This divergence of equilibrium beliefs should the agent withhold information—which, for brevity, we shall refer to as *disagreement*—will prove crucial.[15]

Proposition 1 is stated for an upward biased agent. If the agent is downward biased, he reveals all signals below some threshold, and hence the DM's non-disclosure belief function, $\eta(\hat{s}, p_i, \pi)$, takes the same form as Equation 1 but with $\mathbb{E}_\pi[s|s < \hat{s}]$ replaced by $\mathbb{E}_\pi[s|s > \hat{s}]$. This modified function is single-peaked in $\hat{s}$. If an equilibrium threshold s^* is interior, it satisfies the indifference condition $-\eta(s^*, p_i, \pi) = -s^* - c$. Just as with an upward biased agent, equilibrium is unique when $c \leq 0$ whereas there can be multiple equilibria when $c > 0$. Note that for any $c \neq 0$, the direction of disagreement between the agent and the DM reverses with the direction of agent's bias: for a downward biased agent, $c > 0$ implies that the agent's threshold belief is lower than the DM's nondisclosure belief, and conversely for $c < 0$. Furthermore, when the agent is downward biased, a lower equilibrium threshold intuitively corresponds to revealing less information.

The following comparative statics hold with an upward biased agent; the modifications to account for a downward biased agent are straightforward in light of the above discussion and hence omitted.

Proposition 2. *Assume there is only one agent, and this agent is upward biased.*

1. *An increase in the message cost leads to less disclosure: the highest and lowest equilibrium disclosure thresholds (weakly) increase.*

2. *A higher probability of being informed leads to more disclosure: the highest and lowest equilibrium disclosure thresholds (weakly) decrease.*

The first part of the result is straightforward, as a higher c makes disclosure less attractive. The logic for the second part follows from Lemma 1: given any conjectured threshold, a higher p_i leads to a lower nondisclosure belief, which increases the agent's gain from disclosure over nondisclosure of any signal. For the case of $c = 0$, this comparative static has also been noted by other authors, e.g. by Jung and Kwon (1988) and Acharya et al. (2011).[16] Although we postpone a formal argument to Subsection 3.3, it is worth observing now that these comparative statics have direct welfare implications: with the appropriate comparison of equilibria, a lower message cost and/or a higher probability of the sender being informed (weakly) increases the DM's welfare in a single-agent setting.

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[15]While the agent knows that there is disagreement, the DM does not. Thus, consistent with Aumann (1976), disagreement is not common knowledge in our setting with a common prior.

[16]A related "prejudicial effect" also arises in Che and Kartik (2009), but in their setting the agent conceals signals that are both sufficiently high and sufficiently low because of non-monotonic preferences.
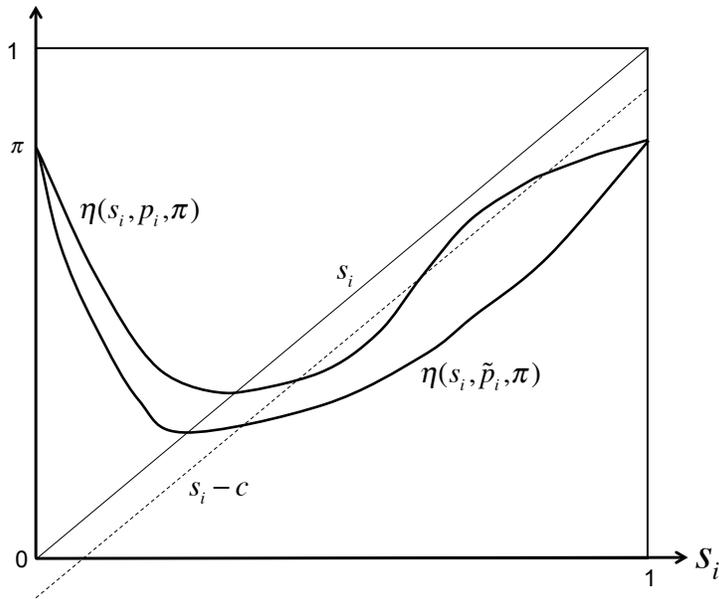
Figure 1: The single-agent game illustrated with parameters $c > 0$ and $\tilde{p}_i > p_i$, $\underline{s} = 0$, $\overline{s} = 1$. Equilibrium disclosure thresholds are given by the intersections of $\eta(s_i, \cdot)$ and $s_i - c$.

Figure 1 summarizes the results of this section.[17]

3.2 Information validates the prior

To study strategic behavior in the multi-agent setting, one must understand how an informed agent expects others' information disclosure to affect the DM's belief, given the agent's signal and his decision to disclose or conceal. First consider disclosure: if an agent i discloses his signal s_i, then the DM's belief based on i's message concedes with i's own belief, viz., s_i. At this "interim" stage, prior to the realization of the other agent j's message, the DM's final posterior is a random variable from the point of agent i (and the DM). No matter what j's equilibrium behavior is, a standard iterated expectations argument implies that agent i's expectation of this random variable is just s_i. Matters are much less straightforward should i conceal his signal s_i. Then, the DM's belief about the state based on i's message will generally be different from i's true signal, as emphasized by the discussion of equilibrium disagreement in the single-agent analysis. Consequently, it is no longer generally true that i's expectation of the DM's final posterior will be i's own belief, s_i. What can be deduced about their relationship?

Any agent's message is an "experiment" in the sense of Blackwell (1951, 1953), with the caveat that because of strategic behavior, the precise nature of the experiment must be determined in equilibrium. The backbone of our analysis is a theorem below that relates garbling in the sense of Blackwell (1951,

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[17]Consider $c > 0$. In the figure, $\eta(\cdot)$ has slope less than one when it crosses $s_i - c$ at the highest crossing point. This makes transparent than an increase in p_i leads to a reduction in the highest equilibrium threshold. If the slope of $\eta(\cdot)$ were larger than one at the highest crossing point, then the highest equilibrium threshold would in fact be $\overline{s}$, and a small increase in p_i would not alter this threshold.

1953) and the expectations of individuals' with different beliefs. To state it precisely for our setting, we recall some standard definitions.[18] An *experiment* is $\mathcal{E} \equiv (Y, \mathcal{Y}, \{P_\omega\}_{\omega=0,1})$, where Y is a measurable space of signals, $\mathcal{Y}$ is a σ-algebra on Y, and each P_ω is probability measure over the signals in state ω. An experiment $\tilde{\mathcal{E}} \equiv (\tilde{Y}, \tilde{\mathcal{Y}}, \{\tilde{P}_\omega\}_{\omega=0,1})$ is a *garbling* of or is *less informative* than experiment $\mathcal{E} \equiv (Y, \mathcal{Y}, \{P_\omega\}_{\omega=0,1})$ if there is a Markov kernel from $(Y, \mathcal{Y})$ to $(\tilde{Y}, \tilde{\mathcal{Y}})$, denoted $P(\cdot|y)$, such that for each $\omega \in \{0, 1\}$ and every set $B \in \tilde{\mathcal{Y}}$, it holds that

$$\tilde{P}_\omega(B) = \int_Y P(B|y)dP_\omega(y).$$

This definition captures the statistical notion that $\tilde{\mathcal{E}}$ does not provide any information that is not contained in $\mathcal{E}$ because signals in the former can be produced by taking signals in the latter and simply transforming them through the kernel $P(\cdot)$; indeed, Blackwell (1951, 1953) used the terminology that $\mathcal{E}$ is "sufficient for" $\tilde{\mathcal{E}}$.

Now, let $\overline{\beta}_m \in (0, 1)$ and $\overline{\beta}_n \in (0, 1)$ denote two individuals' prior beliefs (the probability they each initially hold on state $\omega = 1$). Given any experiment, the individuals' respective priors combine with Bayes rule to determine their respective posteriors, $\beta_m(\cdot)$ and $\beta_n(\cdot)$, where the argument is a signal realization. Let $\mathbb{E}_{\mathcal{E}}^m[\beta_n(\cdot)]$ denote the ex-ante expectation of individual m over the posterior of individual n under experiment $\mathcal{E}$. If $\overline{\beta}_m = \overline{\beta}_n$, then for any $\mathcal{E}$, $\mathbb{E}_{\mathcal{E}}^m[\beta_n(\cdot)] = \overline{\beta}_m$. For individuals with different priors, we can establish the following:

Theorem 1. *Consider two individuals m and n with respective priors $\overline{\beta}_m$ and $\overline{\beta}_n$, and any two experiments $\mathcal{E}$ and $\tilde{\mathcal{E}}$ such that $\tilde{\mathcal{E}}$ is a garbling of $\mathcal{E}$. Then,*

$$\mathbb{E}_{\mathcal{E}}^m[\beta_n(\cdot)] \leq \mathbb{E}_{\tilde{\mathcal{E}}}^m[\beta_n(\cdot)] \iff \overline{\beta}_m \leq \overline{\beta}_n.$$

Furthermore, $\min\{\overline{\beta}_m, \overline{\beta}_n\} \leq \mathbb{E}_{\mathcal{E}}^m[\beta_n(\cdot)] \leq \max\{\overline{\beta}_m, \overline{\beta}_n\}$.

To get some intuition, suppose that individual m is less optimistic than n, i.e. $\overline{\beta}_m < \overline{\beta}_n$. If an experiment $\tilde{\mathcal{E}}$ is completely uninformative—no signal realization would ever change any individual's beliefs—we have $\overline{\beta}_m < \mathbb{E}_{\tilde{\mathcal{E}}}^m[\beta_n(\cdot)] = \overline{\beta}_n$. On the other hand, if an experiment $\mathcal{E}$ is perfectly informative—every signal realization reveals the state—then an individual's prior does not affect his posterior given any signal realization, and so $\overline{\beta}_m = \mathbb{E}_{\mathcal{E}}^m[\beta_m(\cdot)] = \mathbb{E}_{\mathcal{E}}^m[\beta_n(\cdot)] < \overline{\beta}_n$, where the first equality is the previously-noted property of Bayesian updating under any experiment. In other words, individual m believes the perfectly informative experiment will, on expectation, bring individual n's posterior perfectly in line with m's own prior/expected posterior, whereas an uninformative experiment will (obviously) entail no such convergence. (Of course, in turn, individual n expects m to converge to n's prior under the perfectly informative experiment: $\mathbb{E}_{\mathcal{E}}^n[\beta_m(\cdot)] = \overline{\beta}_n$.) Theorem 1 gen-

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[18]The usual version of Blackwell garbling used in economics assumes finite signal spaces (e.g., Crémer, 1982), in which case the definition of garbling can be stated in terms of a Markov or stochastic matrix. Because our setting is one with continuous signals, we need a measure-theoretic definition as in Blackwell (1953).

eralizes this idea to an arbitrary pair of Blackwell-comparable experiments: m anticipates that a more informative experiment will bring, on expectation, n's posterior closer to m's prior. For short, we will say the Theorem shows that, on expectation, *information validates the prior*, or *IVP*.

The key step to formally proving the Theorem is to recognize that, because the individuals are assumed to agree on the experiment—the distribution of signals in each state—and only disagree in their priors over the state, each individual's posterior can be written as a function of both their priors and the other individual's posterior; this observation has also been made by Alonso and Câmara (2013). Specifically, whenever a signal yields individual m a posterior of β_m, it will yield n a posterior of

$$T(\beta_m, \overline{\beta}_n, \overline{\beta}_m) := \frac{\beta_m \frac{\overline{\beta}_n}{\overline{\beta}_m}}{\beta_m \frac{\overline{\beta}_n}{\overline{\beta}_m} + (1 - \beta_m)\frac{1 - \overline{\beta}_n}{1 - \overline{\beta}_m}}. \tag{2}$$

It is straightforward to verify that this posterior transformation mapping, $T(\cdot)$, is concave (resp. convex) in m's posterior when $\overline{\beta}_m < \overline{\beta}_n$ (resp. $\overline{\beta}_m > \overline{\beta}_n$). Theorem 1 follows from an application of Blackwell (1951, 1953), who showed that a garbling increases (resp. reduces) an individual's expectation of any continuous and concave (resp. convex) function of his posterior.[19]

3.3 Main results

We are now ready to study the two-agent disclosure game. For concreteness, we will suppose that both agents are upward biased; the modifications needed when one or both agents are downward biased are straightforward and left to the reader.

Lemma 2. *Any equilibrium is a threshold equilibrium, i.e., both agents use threshold strategies.*

Accordingly, we focus our discussion on threshold strategies. A useful simplification afforded by the assumption of conditionally independent signals is that the DM's belief updating is separable in the agents' messages. In other words, we can treat it as though the DM first updates from either agent i's message just as in a single-agent model, and then using this updated belief like a prior, updates again from the other agent j's message as in a single-agent model without any further attention to i's message. Thus, given any conjectured pair of disclosure thresholds, $(\hat{s}_1, \hat{s}_2)$, there are three relevant nondisclosure beliefs for the DM: if only one agent i discloses his signal, s_i, the DM's belief is $\eta(\hat{s}_j, p_j, s_i)$; if there is nondisclosure from both agents, the DM's belief is $\eta(\hat{s}_j, p_j, \eta(\hat{s}_i, p_i, \pi))$.[20]

In this light, suppose the DM conjectures that agent i is using a disclosure threshold $\hat{s}_i$, and consider whether an agent i should disclose or conceal a signal s_i. As discussed earlier, if i discloses his signal

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[19]Since a garbling induces a mean-preserving contraction in m's posterior (by which we mean the opposite of a mean-preserving spread), and $T(\cdot, \overline{\beta}_n, \overline{\beta}_m)$ is a non-decreasing function for any $\overline{\beta}_n, \overline{\beta}_m$, this step can also understood from the perspective of second order stochastic dominance.

[20]If both agents disclose signals the DM's belief is $\frac{g(s_1, s_2)}{1 + g(s_1, s_2)}$, where $g(s_1, s_2) := \frac{s_1}{1 - s_1} \frac{s_2}{1 - s_2} \frac{1 - \pi}{\pi}$.

then his expectation of the DM's belief—viewed as a random variable that depends on j's message—is s_i, no matter what strategy j is using.[21] On the other hand, if i conceals his signal, then he views the DM as updating from j's message based on a prior of $\eta(\hat{s}_i, p_i, \pi)$ that may be different from s_i. Denote j's disclosure threshold by $\hat{s}_j$, and any particular message sent by j as $m_j \in S \cup \{\phi\}$. Let $\beta(\mathcal{I}; \pi)$ denote the posterior belief given information set $\mathcal{I}$ and prior belief π. Then, agent's i's posterior belief about the state given j's message and his own signal can be written as $\beta(m_j; s_i)$. The posterior transformation mapping from Equation 2 implies that i's expected payoff—equivalently, his expectation of the DM's posterior belief—should he conceal his signal is given by

$$\hat{U}(s_i, \eta(\hat{s}_i, p_i, \pi), \hat{s}_j, p_j) := \mathbb{E}_{\hat{s}_j, p_j}[T(\beta(m_j; s_i), \eta(\hat{s}_i, p_i, \pi), s_i)],$$

where $\mathbb{E}_{\hat{s}_j, p_j}$ denotes that the expectation is taken over m_j using the distribution of beliefs that $\hat{s}_j$ and p_j jointly induce in i about m_j (given s_i).

It is useful to study the "best response" of agent i to any disclosure strategy of agent j. More precisely, let $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ represent the equilibrium disclosure threshold in a (hypothetical) game between agent i and the DM when agent j is conjectured to mechanically adopt disclosure threshold $\hat{s}_j$; we call this agent i's best response. The indifference condition that is necessary and sufficient for an interior best response is

$$U(\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j), p_i, \hat{s}_j, p_j) = \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j) - c, \tag{3}$$

where $U(s_i, p_i, \hat{s}_j, p_j) := \hat{U}(s_i, \eta(s_i, p_i, \pi), \hat{s}_j, p_j)$.[22] In any interior equilibrium of the overall game, (s_1^*, s_2^*), Equation 3 must hold for each agent i when his opponent uses $\hat{s}_j = s_j^*$.

Lemma 3. *For any $\hat{s}_j$ and p_j,*

$$s_i = \eta(s_i, p_i, \pi) \implies U(s_i, p_i, \hat{s}_j, p_j) = s_i,$$
$$s_i > \eta(s_i, p_i, \pi) \implies \eta(s_i, p_i, \pi) \leq U(s_i, p_i, \hat{s}_j, p_j) < s_i,$$
$$s_i < \eta(s_i, p_i, \pi) \implies \eta(s_i, p_i, \pi) \geq U(s_i, p_i, \hat{s}_j, p_j) > s_i.$$

Moreover, both weak inequalities above are strict if and only if $\hat{s}_j < \overline{s}$.

Lemma 3 is a direct consequence of Theorem 1, and it simply reflects that i expects j's information disclosure to, on expectation, bring the DM's nondisclosure belief, $\eta(\cdot)$ closer to s_i. Graphically, this is seen in Figure 2 by comparing the red (short dashed) curve that depicts $U(\cdot)$ with the solid curve depicting $\eta(\cdot)$: the red curve is a rotation of the solid curve around it's fixed point toward the diagonal.

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[21] To be clear: the distribution of the DM's beliefs as a function of j's message depends both on j's strategy and the DM's conjecture about j's strategy. As these two objects must coincide in equilibrium, we will bundle them to ease exposition.

[22] Necessity is clear; sufficiency follows from the argument given in the proof of Lemma 2. Obvious changes apply if the threshold is not interior, which our formal proofs take into account.
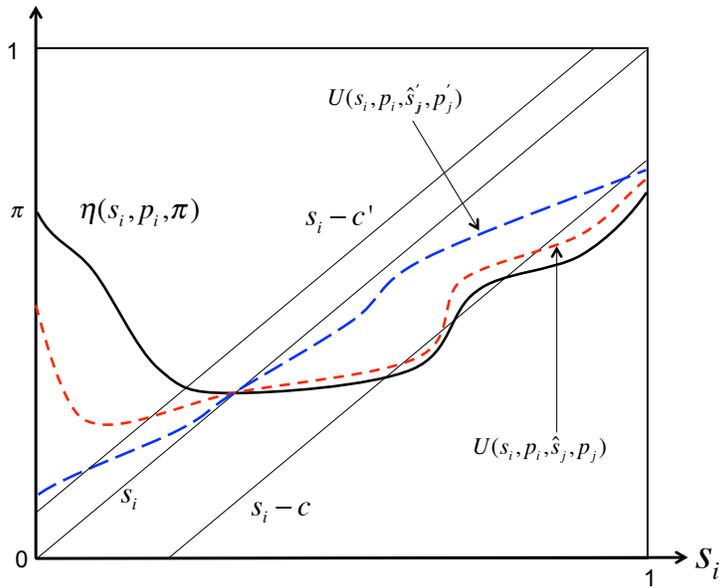
Figure 2: The "best response" of agent i to agent j, illustrated with parameters $c > 0 > c'$, $p'_j > p_j$, and $\hat{s}'_j < \hat{s}_j < \bar{s}$, with $\underline{s} = 0$, $\bar{s} = 1$.

It is evident from the figure that j's information disclosure has very different consequences for the best response of i depending on whether there is a cost of disclosure or a cost of concealment. If $c > 0$ (disclosure cost) then the smallest and largest solutions to Equation 3 will be respectively larger than the smallest and largest single-agent disclosure thresholds; conversely, if $c < 0$ (concealment cost, depicted as c' in Figure 2) the largest solution will be smaller than the unique single-agent threshold;[23] finally, if $c = 0$, the unique solution is the same as the single-agent threshold. These contrasting effects are due to the different natures of disagreement in the single-agent model that we pointed out after Proposition 1. When there is a disclosure cost, the threshold type in any single-agent equilibrium has a *higher* belief than the DM upon nondisclosure, and hence an expected shift of the DM's posterior toward the threshold belief makes concealment more attractive; by contrast, when there is a concealment cost, the threshold type has a *lower* belief than the DM upon nondisclosure, and hence an expected shift of the DM's posterior toward the threshold belief makes concealment less attractive.

Theorem 1 implies that these insights are not restricted to comparisons with the single-agent setting. More generally, the same points hold whenever we compare any $(\hat{s}_j, p_j)$ with $(\hat{s}'_j, p'_j)$ such that the information disclosed by j in the two cases can be ranked in the sense of Blackwell-informativeness. In particular, j's message given $(\hat{s}'_j, p'_j)$ is more informative than his message given $(\hat{s}_j, p_j)$ if $\hat{s}_j \geq \hat{s}'_j$ and $p'_j \geq p_j$; intuitively, the latter scenario has him more likely to be informed and disclosing more conditional on being informed. For a more precise argument, notice that for any message m'_j under

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[23]Although not seen in the figure, there can be multiple solutions to Equation 3 even when $c < 0$.

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$(\hat{s}'_j, p'_j)$, one can garble it to produce message m_j as follows:

$$m_j = \begin{cases} \phi & \text{if } m'_j = \phi \text{ or } m'_j \in [\hat{s}'_j, \hat{s}_j), \\ m'_j \text{ with prob. } \frac{p_j}{p'_j} \text{ and } \phi \text{ with prob. } 1 - \frac{p_j}{p'_j} & \text{if } m'_j \geq \hat{s}_j. \end{cases}$$

In each state of the world, the distribution of m_j as just constructed is the same as the distribution of agent j's message under $(\hat{s}_j, p_j)$. Thus, the message produced under $(\hat{s}_j, p_j)$ is a garbling of that produced under $(\hat{s}'_j, p'_j)$.

The effect of agent j's message becoming more informative is depicted in Figure 2 as a shift from the red (short dashed) curve to the blue (long dashed) curve. Plainly, whether agent i's best response is to disclose less or more of his own information turns on whether $c > 0$ or $c < 0$; the logic is the same as discussed earlier. Three key points emerge: information disclosure between the two agents are *strategic substitutes* when there is a disclosure cost, *strategic complements* when there is a concealment cost, and *independent* when there is no message cost.

For any given $\hat{s}_j$, there can be multiple solutions to Equation 3, hence $\hat{s}_i^{BR}(\cdot)$ is a best-response correspondence. We say that agent i's best response increases if the largest and the smallest element of $\hat{s}_i^{BR}(\cdot)$ both increase (weakly); we say that the best response is strictly greater than some $\hat{s}$, written $\hat{s}_i^{BR}(\cdot) > \hat{s}$, if the smallest element of $\hat{s}_i^{BR}(\cdot)$ is strictly greater than $\hat{s}$.

Proposition 3. *Assume both agents are upward biased. Any agent i's best-response disclosure threshold $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ is decreasing in p_i. Furthermore, letting $\hat{s}_i^0$ denote the unique (resp. smallest) equilibrium threshold in the single-agent game with i when $c \leq 0$ (resp. $c > 0$):*

1. *(Independence). If $c = 0$, then $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j) = \hat{s}_i^0$ independent of $\hat{s}_j$ and p_j.*

2. *(Strategic complements). If $c < 0$, then (i) $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j) \leq \hat{s}_i^0$, with equality if and only if $\hat{s}_i^0 = \underline{s}$ or $\hat{s}_j = \overline{s}$, and (ii) $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ is increasing in $\hat{s}_j$ and decreasing in p_j.*

3. *(Strategic substitutes). If $c > 0$, then (i) $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j) \geq \hat{s}_i^0$, with equality if and only if $\hat{s}_i^0 = \overline{s}$ or $\hat{s}^j = \overline{s}$, and (ii) $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ is decreasing in $\hat{s}_j$ and increasing in p_j.*

Since each agent's best response is monotone, existence of an equilibrium in the two-agent game follows from Tarski's fixed point theorem. When $c < 0$ (concealment cost), the strategic complementarity in disclosure thresholds implies that there is a largest equilibrium, which corresponds to the highest equilibrium thresholds for both agents. For reasons discussed earlier, each agent's message in the largest equilibrium is a garbling of his message in any other equilibrium. It follows that the largest equilibrium is the least informative and the *worst* in terms of the DM's welfare. Similarly, the smallest equilibrium is the most informative and the *best* in terms of the DM's welfare. On the other hand, when $c > 0$ (disclosure cost), the two disclosure thresholds are strategic substitutes. There is an *i-maximal* equilibrium that maximizes agent i's threshold and also minimizes agent j's threshold

across all equilibria. Likewise, there is a *j-maximal* equilibrium that minimizes agent i's threshold and also maximizes agent j's threshold across all equilibria. These two equilibria are not ranked in terms of Blackwell-informativeness and in general cannot be welfare ranked for the DM; moreover, neither of these equilibria may correspond to either the best or worst equilibrium for the DM.[24]

The following result is derived using standard monotone comparative statics arguments.

Proposition 4. *Assume both agents are upward biased. For any $i \in \{1, 2\}$:*

1. *If $c \leq 0$, then an increase in p_i or a decrease in c weakly lowers the disclosure thresholds of both agents in both the worst and the best equilibria, and weakly increases the DM's welfare in these equilibria.*

2. *If $c > 0$, then an increase in p_i weakly lowers agent i's disclosure threshold and weakly raises agent j's disclosure threshold in both the i-maximal and the j-maximal equilibria. A decrease in c has ambiguous effects on the two agents' equilibrium disclosure thresholds in both the i- and j-maximal equilibria. Furthermore, an increase in p_i or a decrease in c can strictly reduce the DM's welfare in the DM's best equilibrium.*

One can view the single-agent game with i as a two-agent game where agent j is never informed, i.e., $p_j = 0$. With this in mind, a comparison of the single-agent game and the two-agent game can be obtained as an immediate corollary to Proposition 3 and Proposition 4.

Corollary 1. *Assume both agents are upward biased and let $\hat{s}_i^0$ denote the unique (resp. smallest) equilibrium threshold in the single-agent game with i when $c \leq 0$ (resp. $c > 0$).*

1. *If $c = 0$, equilibrium in the two-agent game is unique and is equal to $(\hat{s}_1^0, \hat{s}_2^0)$. Hence, the DM's welfare is strictly higher in the two-agent game than in a single-agent game with either agent.*

2. *If $c < 0$, every equilibrium in the two-agent game is weakly smaller than $(\hat{s}_1^0, \hat{s}_2^0)$, with equality if and only if $\hat{s}_1^0 = \hat{s}_2^0 = \underline{s}$. Hence, the DM's welfare is strictly higher in any equilibrium of the two-agent game than in a single-agent game with either agent.*

3. *If $c > 0$, every equilibrium in the two-agent game is weakly larger than $(\hat{s}_1^0, \hat{s}_2^0)$, with equality if and and only if $\hat{s}_1^0 = \hat{s}_2^0 = \overline{s}$. The DM's welfare in the best equilibrium of the two-agent game may be higher or lower than in the best equilibrium of the single-agent game with agent i or agent j alone.*

Part 1 of the corollary follows from Part 1 of Proposition 3. When $c = 0$, the best response of each agent is to use the same disclosure threshold as in the single-agent setting, regardless of the other agent's strategy. Since the DM receives two messages instead of just one, and the probability

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[24]The agents' welfare ranking across equilibria is straightforward, because, as explained after Proposition 1, this just depends on the probability of disclosure. When $c < 0$, both agents' welfare is lowest in the largest equilibrium and highest in the smallest equilibrium; when $c > 0$, each agent i's welfare is highest in the i-maximal equilibrium and lowest in the j-maximal equilibrium.

distribution of these messages remain the same in the single-agent game, she is better off when facing both agents than when facing either agent alone.

Part 2 of Corollary 1 can be obtained by considering the worst equilibrium of the two-agent game. When $c < 0$, let $\mathbf{s}^*(p_i, p_j)$ represent the vector of disclosure thresholds in the worst equilibrium. Proposition 4 implies that $s_i^*(p_i, p_j) \leq s_i^*(p_i, 0) = \hat{s}_i^0$ and $s_j^*(p_i, p_j) \leq s_j^*(0, p_j) = \hat{s}_j^0$. Thus $\mathbf{s}^*(p_i, p_j)$ is weakly smaller than $(\hat{s}_i^0, \hat{s}_j^0)$, and hence every equilibrium is weakly smaller than $(\hat{s}_i^0, \hat{s}_j^0)$. It follows that the DM's welfare is strictly higher than in the unique equilibrium of the single-agent game with either agent. This higher welfare is due to both a direct effect of receiving information from an additional agent, as in the case of $c = 0$, but generally also due to an indirect effect wherein each agent is now more informative than in the single-agent setting.

Finally, for the case of disclosure cost ($c > 0$), let $\mathbf{s}^{*,i}(p_i, p_j)$ represent the i-maximal equilibrium and $\mathbf{s}^{*,j}(p_i, p_j)$ represent the j-maximal equilibrium. Since the latter also minimizes i's disclosure threshold, it follows that in any equilibrium, i's threshold is at least as large as $s_i^{*,j}(p_i, p_j) \geq s_i^{*,j}(p_i, 0) = \hat{s}_i^0$, where the inequality is by Part 2 of Proposition 4. Analogously, agent j's threshold in any equilibrium is at least as large as $s_j^{*,i}(p_i, p_j) \geq s_j^{*,i}(0, p_j) = \hat{s}_j^0$. Thus, in any equilibrium of the two-agent game, both agents are (weakly) less informative than in the DM's best equilibrium of the single-agent game. The overall welfare comparison between the two-agent game and the single-agent game is generally ambiguous: while adding a second agent has a direct effect of increasing the DM's information, there is an adverse indirect effect due to strategic substitution that makes the original agent disclose less.

The following example shows that the net effect can be strictly negative for the DM's welfare across the best equilibria, even when the two agents have opposite biases, which is a scenario that is often thought to particularly promote information disclosure.

Example 1. *The prior is $\pi = 1/2$. The information structure is parametrized by $\gamma \in (1/2, 1)$ and $\delta \in (0, 1)$. There are four possible signal realizations,[25] with conditional probabilities $\Pr(s|\omega)$ given as follows:*

	$\underline{s} = 0$	$s^l = 1 - \gamma$	$s^h = \gamma$	$\overline{s} = 1$
$\omega = 0$	$1 - \delta$	$\gamma\delta$	$(1 - \gamma)\delta$	0
$\omega = 1$	0	$(1 - \gamma)\delta$	$\gamma\delta$	$1 - \delta$

The DM must choose an action $a \in \mathbb{R}$ and her von Neumann-Morgenstern preferences are represented by a quadratic loss function: $u_{DM}(a, \omega) = -(a - \omega)^2$.

In this setting, there is an open and dense set of parameters $(c, p_1, p_2, \gamma, \delta)$ with $c > 0$, such that:

- *With a single upward biased agent, the DM's best equilibrium has the agent only disclosing signals s^h and $\overline{s}$ (and symmetrically if the agent is downward biased).*

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[25]While this formally violates our assumption of continuous signals, recall fn. 13; one can also perturb the example to make it continuous without affecting the conclusion.

- *With two upward biased agents, the DM's best equilibrium has each agent only disclosing signal $\bar{s}$. The DM's welfare in this equilibrium is strictly lower than in the above single-agent equilibrium.*

- *With opposite biased agents, the DM's best equilibrium has the upward biased agent only disclosing signal $\bar{s}$ and the downward biased agent only disclosing signal $\underline{s}$. The DM's welfare in this equilibrium is strictly lower than in the best equilibrium with either agent alone.*

Furthermore, given agents with opposite biases, there is an open and dense set of parameters such that an increase in the disclosure cost $c > 0$ strictly raises the DM's welfare in her best equilibrium.

The calculations verifying these claims are provided in the Supplementary Appendix.

It is appropriate to compare our welfare results with Bhattacharya and Mukherjee (2013). As noted in fn. 12, they study a related model to ours but maintain $c = 0$ and assume perfectly correlated signals. They show that an increase in the probability of an agent being informed can reduce the DM's welfare (which they assume is a quadratic loss function); however, as seen in their Corollary 2, a necessary condition for this to happen in their model is that at least one agent must have non-monotonic preferences over the DM's posterior, which in turn implies (because agents have single-peaked preferences) that the agents share the same ranking over decisions on a subset of the decision space. In this sense, their result requires agents who are not in "pure conflict", unlike in our Example 1. More broadly, our results on equilibrium behavior and welfare for $c \neq 0$ are orthogonal and complementary to their treatment of non-monotonic preferences.

4   Generalizations and Caveats

4.1   Many agents

Our results readily generalize to any finite number of agents. Suppose in addition to agents i and j, there are K other agents, all of whom simultaneously send messages to the DM. Let $\mathbf{m}$ represent the collection of these K messages. Then, agent i's posterior belief given his own signal s_i, agent j's message m_j, and the K other agents' messages $\mathbf{m}$ is $\beta(m_j, \mathbf{m}; s_i) = \beta(m_j; \beta(\mathbf{m}; s_i))$. The DM's belief given the K agents' messages $\mathbf{m}$ and given non-disclosure by agent i is $\eta(\hat{s}_i, p_i, \beta(\mathbf{m}; \pi))$. Thus, the posterior transformation mapping from Equation 2 and the law of iterated expectations implies that the expected payoff for agent i from concealing his signal is

$$\mathbb{E}\left[\mathbb{E}_{\hat{s}_j, p_j}[T(\beta(m_j; \beta(\mathbf{m}; s_i)), \eta(\hat{s}_i, p_i, \beta(\mathbf{m}; \pi)), \beta(\mathbf{m}; s_i)) \mid \mathbf{m}]\right].$$

The inside expectation in the above expression is taken over the distribution of m_j, while the outside expectation is taken over the distribution of $\mathbf{m}$ generated from the equilibrium strategies of the K

agents. Given any $\mathbf{m}$, the transformation $T(\cdot)$ in the multi-agent case is the same as that in the two-agent case, with the common prior π replaced by $\beta(\mathbf{m}; \pi)$. Since our results hold for any π, the logic of strategic substitution or strategic complementarity continues to apply in the multiple-agent case. In particular, when $c < 0$, $\mathbb{E}_{\hat{s}_j, p_j}[T(\cdot) \mid \mathbf{m}]$ increases in $\hat{s}_j$ and decreases in p_j for any $\mathbf{m}$. Consequently, agent i's expected payoff from non-disclosure, $\mathbb{E}\left[\mathbb{E}_{\hat{s}_j, p_j}[T(\cdot) \mid \mathbf{m}]\right]$ also increases in $\hat{s}_j$ and decreases in p_j. Thus disclosure by any two agents are strategic complements. Similarly, in the case of disclosure cost (i.e., $c > 0$), disclosure by any two agents are strategic substitutes.

It follows from these observations that when there is either no message cost or a concealment cost ($c \leq 0$), the DM always benefits from having more agents to supply her with information. When there is disclosure cost ($c > 0$), on the other hand, an increase in the number of agents has ambiguous effects on each agent's disclosure threshold, and can lead to either an increase or decrease in the DM's welfare (in the best equilibrium for the DM).

4.2 Many states

We next turn to the generalization of our results to more than two states. Consider an arbitrary finite number of states, $\Omega = \{\omega_0, \ldots, \omega_L\}$, where each $\omega_l \in \mathbb{R}$, and without loss of generality, $\omega_l < \omega_{l+1}$ for all $0 \leq l < L$ (with $L \geq 1$). We will now use bold symbols to denote probability distributions over Ω viewed as vectors in $\mathbb{R}^{L+1}$.

4.2.1 Information validates the prior

Our first task is to identify conditions under which information validates the prior. Following standard terminology, we say that a belief $\boldsymbol{\beta}'$ *likelihood-ratio dominates* belief $\boldsymbol{\beta}$, written $\boldsymbol{\beta}' \geq_{LR} \boldsymbol{\beta}$, if

$$\forall l \in \{1, \ldots, L\} : \ \beta'(\omega_l)\beta(\omega_{l-1}) \geq \beta'(\omega_{l-1})\beta(\omega_l).$$

An experiment is now $\mathcal{E} \equiv (Y, \mathcal{Y}, \{P_\omega\}_{\omega \in \Omega})$. We restrict attention to experiments where the probability measure in each state, P_ω, is absolutely continuous with respect to some reference measure on $(Y, \mathcal{Y})$, and let $p(\cdot|\omega)$ denote the Radon-Nikodym derivative with respect to that reference measure.[26] We denote posterior beliefs given realization y as $\boldsymbol{\beta}(y) \equiv (\beta(\omega_0|y), \ldots, \beta(\omega_L|y))$.

Definition 1. An experiment $\mathcal{E} \equiv (Y, \mathcal{Y}, \{P_\omega\}_{\omega \in \Omega})$ is an *MLRP-experiment* if there is a total order on Y, denoted $\geq$ (with asymmetric relation $>$), such that the monotone likelihood ratio property holds:

$$y' > y \text{ and } \omega' > \omega \implies p(y'|\omega')p(y|\omega) \geq p(y'|\omega)p(y|\omega').$$

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[26]Of course, this does not imply that for any ω and ω', P_ω is absolutely continuous with respect to $P_{\omega'}$; hence, we are not ruling out perfectly revealing signals.

As is well known, this weak form of monotone likelihood ratio property (MLRP) is without loss of generality when there are only two states, hence any experiment is an MLRP-experiment when $L = 1$.

To state the generalization of Theorem 1, for any belief $\boldsymbol{\beta}$ let $M(\boldsymbol{\beta}) := \sum_{\omega \in \Omega} \omega \beta(\omega)$ be the expectation of the state given $\boldsymbol{\beta}$.

Theorem 2. *Assume any finite number of states. Consider two individuals m and n with respective priors $\overline{\boldsymbol{\beta}}_m$ and $\overline{\boldsymbol{\beta}}_n$ such that either $\overline{\boldsymbol{\beta}}_m \geq_{LR} \overline{\boldsymbol{\beta}}_n$ or $\overline{\boldsymbol{\beta}}_n \geq_{LR} \overline{\boldsymbol{\beta}}_m$, and any two MLRP-experiments $\mathcal{E}$ and $\tilde{\mathcal{E}}$ such that $\tilde{\mathcal{E}}$ is a garbling of $\mathcal{E}$. Then,*

$$\mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))] \leq \mathbb{E}_{\tilde{\mathcal{E}}}^m[M(\boldsymbol{\beta}_n(\cdot))] \iff M(\overline{\boldsymbol{\beta}}_m) \leq M(\overline{\boldsymbol{\beta}}_n).$$

Furthermore, $\min\{M(\overline{\boldsymbol{\beta}}_m), M(\overline{\boldsymbol{\beta}}_n)\} \leq \mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))] \leq \max\{M(\overline{\boldsymbol{\beta}}_m), M(\overline{\boldsymbol{\beta}}_n)\}.$

A key step in proving this theorem is that, analogous to the two-state case, the entire posterior belief distribution of individual n can be derived from the posterior distribution of individual m and the two individuals' priors. Specifically, given any experiment, the posterior of an individual n on state ω after signal realization y, $\beta_n(\omega|y)$, can be written as a function of individual m's posterior on ω, $\beta_m(\omega|y)$, and the two individuals' respective prior beliefs, $\overline{\boldsymbol{\beta}}_n$ and $\overline{\boldsymbol{\beta}}_m$:

$$\beta_n(\omega|y) = \frac{\beta_m(\omega|y)\frac{\overline{\beta}_n(\omega)}{\overline{\beta}_m(\omega)}}{\sum_{\omega' \in \Omega} \beta_m(\omega'|y)\frac{\overline{\beta}_n(\omega')}{\overline{\beta}_m(\omega')}}.$$

Accordingly, as we are interested in the posterior expectation, the generalization of transformation in Equation 2 is given by the following function:

$$\mathcal{T}(\boldsymbol{\beta}_m, \overline{\boldsymbol{\beta}}_n, \overline{\boldsymbol{\beta}}_m) := \frac{\sum_{\omega} \omega \beta_m(\omega)\frac{\overline{\beta}_n(\omega)}{\overline{\beta}_m(\omega)}}{\sum_{\omega'} \beta_m(\omega')\frac{\overline{\beta}_n(\omega')}{\overline{\beta}_m(\omega')}}. \tag{4}$$

Unlike in the two-state case, the function $\mathcal{T}(\cdot, \overline{\boldsymbol{\beta}}_n, \overline{\boldsymbol{\beta}}_m)$ is not generally convex (resp. concave) in the first argument even when $\overline{\boldsymbol{\beta}}_n \geq_{LR} \overline{\boldsymbol{\beta}}_n$ (resp. $\overline{\boldsymbol{\beta}}_n \leq_{LR} \overline{\boldsymbol{\beta}}_m$). Thus, one cannot apply the results of Blackwell (1953) as we did to prove Theorem 1. Instead, our proof relies on more direct argument.

Remark 1. The likelihood-ratio ordering assumptions in Theorem 2 are tight in the following sense: (i) there exist priors $\overline{\boldsymbol{\beta}}_m \not\geq_{LR} \overline{\boldsymbol{\beta}}_n$ and an MLRP-experiment $\mathcal{E}$ such that $M(\overline{\boldsymbol{\beta}}_m) > M(\overline{\boldsymbol{\beta}}_n) > \mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))]$; and (ii) there exist priors $\overline{\boldsymbol{\beta}}_m >_{LR} \overline{\boldsymbol{\beta}}_n$ and a non-MLRP-experiment $\mathcal{E}$ such that $M(\overline{\boldsymbol{\beta}}_m) > M(\overline{\boldsymbol{\beta}}_n) > \mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))]$. See the Supplementary Appendix for examples proving both these claims.

4.2.2 Applying IVP

Recall that in our strategic disclosure setting, the signal structure is given by a family of cumulative distributions $\{F(s|\omega)\}_{\omega \in \Omega}$ with corresponding densities $\{f(s|\omega)\}_{\omega \in \Omega}$. To avoid some technical details, we assume now that for all ω, $f(s|\omega)$ is continuous in s and strictly positive on $[\underline{s}, \overline{s}] \subseteq [0, 1]$. We no longer equate signals with private beliefs; rather, there is a full-support common prior, $\pi \in \Delta(\Omega)$, and any agent's belief under signal s is denoted $\boldsymbol{\beta}(s) = (\beta(\omega_0|s), \ldots, \beta(\omega_L|s))$, where Bayes rule implies that for each ω,

$$\beta(\omega|s) = \frac{f(s|\omega)\pi(\omega)}{\sum_{\tilde{\omega} \in \Omega} f(s|\tilde{\omega})\pi(\tilde{\omega})}. \tag{5}$$

As earlier, we focus here on two agents who are both upward biased, leaving the modifications for other combinations of biases to the reader.

Theorem 2 can be applied to our strategic disclosure problem if (i) the endogenously-generated experiments—viz., equilibrium information disclosure by each agent—fall in the class of MLRP-experiments, and (ii) when an agent uses a threshold strategy, the agent's belief at the threshold and the DM's nondisclosure belief can be likelihood-ratio ranked. To this end, we assume:

Assumption 1. *The information structure is such that*

1. *The strict MLRP holds in the signals and states, i.e.*

$$\forall s' > s \text{ and } \forall \omega' > \omega : f(s'|\omega')f(s|\omega) > f(s'|\omega)f(s|\omega').$$

2. *For each agent i and any two signals, s' and $s \leq s'$,*

$$\frac{1 - p_i + p_i F(s|\omega)}{f(s'|\omega)}$$

   is weakly monotone (increasing or decreasing) in ω.

The first part of the assumption is familiar and ensures that for any s' and s, $\boldsymbol{\beta}(s)$ and $\boldsymbol{\beta}(s')$ are likelihood-ratio ordered. To understand the second part, denote the DM's nondisclosure belief when the upward biased agent i uses a threshold s_i as $\boldsymbol{\eta}(s_i, p_i) = (\eta(\omega_0|s_i, p_i), \ldots, \eta(\omega_L|s_i, p_i))$.[27] Bayes rule implies that for each ω,

$$\eta(\omega|s_i, p_i) = \frac{(1 - p_i + p_i F(s_i|\omega))\pi(\omega)}{1 - p_i + p_i \sum_{\tilde{\omega} \in \Omega} F(s_i|\tilde{\omega})\pi(\tilde{\omega})}. \tag{6}$$

Agent i's message can be viewed as an MLRP-experiment if every pair of his messages are likelihood-ratio ordered. Given any disclosure threshold s_i, this is assured if for all $s'_i \geq s_i$ (as the agent only

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[27] For brevity, we suppress the dependence on π.

discloses signals above the threshold), $\eta(\omega|s_i, p_i)/\beta(\omega|s_i')$ is weakly monotone in ω. Inspecting Equation 5 and Equation 6, this is precisely the second part of Assumption 1.

Lemma 4. *Assume Assumption 1. Then, for any i, s_i and $s_i' \geq s_i$, either $\boldsymbol{\eta}(s_i, p_i) \geq_{LR} \boldsymbol{\beta}(s_i')$ or $\boldsymbol{\eta}(s_i, p_i) \leq_{LR} \boldsymbol{\beta}(s_i')$. Consequently, if i uses a threshold strategy, then i's message represents an MLRP-experiment.*

Plainly, Assumption 1 is demanding when (and only when) there are more than two states; in particular, the second part implies that for any s, $f(s|\omega)$ must be weakly monotone in ω (as seen by taking $s = \underline{s}$ in the requirement), which given strict MLRP, is equivalent to the existence of a neutral or uninformative signal in the sense of Milgrom (1981, p. 384).[28] Nevertheless, there are known parametric families that satisfy Assumption 1 for any p_i.

Example 2. *Let $S = [0, 1]$ and fix any two functions $g : \Omega \to \mathbb{R}_{++}$ and $h : \Omega \to \mathbb{R}_{++}$ such that $\frac{g(\omega)}{h(\omega)}$ is strictly increasing in ω. Let*

$$f(s|\omega) = \frac{2[sg(\omega) + (1-s)h(\omega)]}{g(\omega) + h(\omega)}. \tag{7}$$

The Supplementary Appendix verifies that this signal structure satisfies Assumption 1 for any p_i. When $g(\omega) = 1 - h(\omega)$ and $h(\omega) = \frac{2-a\omega}{4}$ for some $a \in \left(0, \frac{2}{\max|\omega|}\right)$ (which ensures all the required conditions), Equation 7 reduces to $f(s|\omega) = 1 + a\left(s - \frac{1}{2}\right)\omega$, which is a Farlie-Gumbel-Morgenstern linear copula and (up to normalizations) is the multiplicative linear structure used in Ottaviani and Sørensen (2006) (see also Piccione and Tan, 1996).

Proposition 5. *Assume Assumption 1. In any threshold equilibrium, each agent i's threshold, s_i^*, satisfies:*

1. $c = 0 \implies \boldsymbol{\beta}(s_i^*) = \boldsymbol{\eta}(s_i^*, p_i)$.

2. $c < 0 \implies \boldsymbol{\beta}(s_i^*) <_{LR} \boldsymbol{\eta}(s_i^*, p_i)$.

3. $c > 0 \implies \boldsymbol{\beta}(s_i^*) >_{LR} \boldsymbol{\eta}(s_i^*, p_i)$;

Proposition 5 generalizes the ideas from Section 3 that in a comparison of the agent's equilibrium threshold belief with the DM's nondisclosure belief, (i) the two are equal when there is no message cost, (ii) the former is strictly lower than the latter (in the sense of likelihood-ratio dominance) when there is a concealment cost, and (iii) the former is strictly higher than the latter when there is a disclosure cost. Since Lemma 4 ensures that Theorem 2 can applied when agents use threshold strategies, IVP implies that there is a strategic complementarity (resp. substitutability) in disclosure behavior when $c < 0$ (resp. $c > 0$), reasoning as earlier. As the essence of the logic is the same as that explained in Section 3, in the interest of space we do not provide a formal statement of the analog of Proposition 3.

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[28]More precisely, $\exists s \in (\underline{s}, \overline{s})$ such that $\forall \omega', \omega, f(s|\omega') = f(s|\omega)$.

4.3 Sequential reporting

Return now to the two-state setting, for simplicity. The key insight from our analysis of the multi-sender game with simultaneous disclosure extends to a setting where agents disclosure sequentially. For concreteness, consider a two-sender game in which both agents are upward biased but disclosure is sequential: agent 1 reports first and his message m_1 is made public to both the DM and agent 2 before agent 2 submits his report. Agent 2 now effectively faces a single-agent problem where he and the DM share a common prior, say $\beta(m_1; \pi)$, which is a function to be determined in equilibrium. Proposition 1 implies that Agent 2 will adopt the disclosure threshold $s^*(p_2, \beta(m_1; \pi))$.

Consider now the disclosure decision of Agent 1 when the DM conjectures that he is using a non-disclosure threshold $\hat{s}_1$, with corresponding nondisclosure belief $\eta(\hat{s}_1, p_1, \pi)$. If Agent 1 discloses his signal s_1, his expectation of the DM's posterior belief is simply s_1. Following the notation used in Subsection 3.3 (in particular, p. 13) if Agent 1 chooses non-disclosure, his expectation of the DM's posterior belief is

$$\mathbb{E}_{s^*(p_2, \beta(\phi; \pi)), p_2}[T(\beta(m_2; s_1), \eta(\hat{s}_1, p_1, \pi), s_1)].$$

Since Agent 2 discloses more when he is better informed (Proposition 2), a higher p_2 makes the message m_2 more Blackwell-informative, both directly through a higher probability of agent 2 getting a signal and indirectly through a lower disclosure threshold. That information validates the prior implies that the DM's belief is expected to move away from $\eta(\hat{s}_1, p_1, \pi)$ toward s_1. The same logic that establishes Proposition 4 therefore gives the following result, whose proof is omitted.

Proposition 6. *Consider sequential disclosure and assume Agent 1, the first mover, is upward biased. If $c > 0$ (resp. $c < 0$), a higher p_2 weakly lowers (resp. weakly raises) the equilibrium disclosure threshold of Agent 1 in the 1-maximal and 1-minimal equilibria.*

An immediate corollary to Proposition 6 is that, in the case of concealment cost, the first agent discloses more than he does in a single-agent setting. As a result, the DM is always better off in a sequential game than with Agent 1 alone. On the other hand, a welfare comparison between the sequential game and the simultaneous move game is generally ambiguous.

4.4 Conditional correlation

The assumption that agents' signals are conditionally independent is clearly important for our analytical methodology, as without it we cannot apply Theorem 1. While relaxing this in general appears intractable, we can illustrate how some of our substantive conclusions would change under a significantly different information structure.

Consider the extreme case where informed agents' signals are perfectly correlated. In other words, there is single signal s drawn from a distribution $F(s|\omega)$ and each agent i is then independently either

informed of s with probability p_i or remains uninformed. When $c = 0$ the strategic setting is effectively identical to the "extreme agenda" case of Bhattacharya and Mukherjee (2013).[29] If both agents are biased in the same direction then this model can be mapped to a single-agent problem where the agent is informed with probability $p_i + p_j - p_i p_j$, which is larger than $\max\{p_i, p_j\}$.[30] Proposition 2 then implies that for any value of c, each agent discloses more when there is an additional agent; hence, the DM is always better off with two agents than one (in both the best and worst equilibria).

To understand the contrast with our earlier results, it is instructive to consider in particular why the irrelevance result no longer holds for $c = 0$. For simplicity, suppose both agents are upward biased and symmetric ($p_i = p_j = p$). Let s^* denote the common single-agent threshold, so that the nondisclosure belief satisfies $\eta(s^*, p, \pi) = s^*$. The essential observation is that when agent j is added to the picture, say with the hypothesis that he too discloses all signals weakly above s^*, type s^* of agent i no longer expects the DM's belief to be s^* should he conceal his signal, in contrast to the case of conditionally independent signals. Rather, he expects the DM's belief to be strictly lower: if j is informed the DM's belief will be s^*, and if j is uninformed the DM's belief will be strictly lower because of nondisclosure from two agents rather than just one. This makes type s^* of agent i strictly prefer disclosure. From the perspective of Theorem 1, the point is that under conditionally correlated signals, when an informed agent i does not disclose his signal, i and the DM do not agree on the experiment generated by j's message; thus, even if the DM's nondisclosure belief agrees with i's belief (over the state), i's expectation of the DM's posterior belief can be different.

Interestingly, welfare conclusions under perfectly correlated signals are very different when the agents are opposite biased. For simplicity continue to consider $c = 0$. Proposition 8 in the Appendix establishes that when agents are opposite biased, each agent discloses *strictly less* than he does in his single-agent game. Thus, despite the increased availability of information, the overall disclosure of information in the two-agent setting is not (Blackwell) more informative than under either single-agent problem. Consequently, for either agent i, there exist preferences of the DM such that she would strictly prefer to face agent i alone rather than the two agents simultaneously. An implication is that the welfare conclusion in Corollary 2 of Bhattacharya and Mukherjee (2013) is sensitive to the preference structure they assume for the DM.

To summarize this discussion:

Remark 2. Under perfectly correlated signals and no message cost, competition between two opposite-biased agents can make the DM worse off (relative to a single-agent problem) whereas competition between two like-biased agents always makes the DM better off.

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[29]Note that they allow for the agents' utility functions to be non-linear; the ensuing discussion does not depend on linearity either because our single-agent analysis does not require linearity.

[30]To see this, observe first that perfect correlation implies that there is only one relevant nondisclosure belief, viz. when both agents don't disclose. So agents who are biased in the same direction must use the same equilibrium disclosure threshold. Given any such threshold, the nondisclosure belief is then computed just as in Equation 1 (assuming the bias is upward), but where $1 - p$ is now the probability that both agents are uninformed, i.e. $(1 - p_i)(1 - p_j)$.

4.5 Preferences

Another assumption that is important in applying Theorem 1 is that each agent has linear preferences. Suppose, more generally, that an agent i's utility is given by some function, $V_i(\beta_{DM})$. The comparative statics of agent i's disclosure depends on the comparative statics of

$$\mathbb{E}[V_i(T(\beta_i, \overline{\beta}_i, \overline{\beta}_{DM}))] - \mathbb{E}[V_i(\beta_i)] \tag{8}$$

across Blackwell-comparable experiments, where the expectation is taken over the posterior β_i using i's beliefs and $T(\cdot)$ is the transformation of Equation 2. When $V_i(\cdot)$ is linear, one can ignore the second term in (8) as it is constant across experiments and Theorem 1 tells us that the sign of the change in the first term is determined by the sign of $\overline{\beta}_i - \overline{\beta}_{DM}$. While unambiguous comparative statics of (8) cannot be obtained for arbitrary specifications, it is straightforward that because $T(\beta_i, x, x) = \beta_i$ for any β_i and x, the logic behind our irrelevance result extends generally. In particular:

Proposition 7. *If $c = 0$ and $V_i(\cdot)$ is strictly monotone, then no matter j's disclosure strategy, the best response disclosure threshold for i is the same as when he is a single agent.*

When $c \neq 0$, there are non-linear specifications for $V_i(\cdot)$ under which our themes about strategic complementarity under concealment cost or substitutability under disclosure cost do extend, and there are other specifications which make conclusions ambiguous or even reversed. The Supplementary Appendix illustrates these points using a parametric family of exponential utility functions.

5   Concluding Remarks

We have studied the disclosure of conditionally-independent verifiable signals by multiple agents who each have linear preferences over a DM's posterior expectation of a state variable. We have shown that when there is neither a cost of disclosure nor of concealing information ($c = 0$), the presence of other agents is strategically irrelevant for any agent. On the other hand, agents' disclosure are strategic complements under concealment costs ($c < 0$), whereas they are strategic substitutes under disclosure costs ($c > 0$). Consequently, a DM is always better off when more agents are available—or the message cost is lower, or any agent is more likely to be informed—when $c \leq 0$ whereas any of these changes in the environment can strictly harm the DM when $c > 0$.

These findings derive from a general result about how Bayesian individuals with mutually-known differing priors over a state variable expect more information in the sense of Blackwell (1951, 1953) to further validate their own priors: each believes that more information will, on expectation, drive the other's posterior (expectation) closer to her own prior (expectation). Besides its application to our current framework, we believe this is a result of intrinsic interest and it can be fruitfully applied in other contexts as well.

It bears emphasis that our comparative statics results for strategic disclosure hold regardless of whether agents are like-biased or opposed in terms of which direction they would like to influence the DM in. Moreover, while we have assumed that the message cost is common to all agents, this is not important. If costs were instead agent-specific, then the effect of a change in other agents' disclosure on agent i's disclosure would turn solely on i's personal message cost. The reason is that the direction of equilibrium disagreement between agent i's threshold type and the DM depends only on i's message cost. Both these points clarify that the strategic substitutes property under disclosure cost should not be viewed from a perspective of disclosure being a costly public good on which agents free ride. Rather, any agent's expected payoff difference between disclosure and concealment is reduced when there is more information disclosure by other agents.

We have focussed on the strategic interplay of disclosure between multiple agents. However, our analysis is also directly applicable to other related issues. For example, suppose there is only one strategic agent but the DM also (simultaneously) receives some exogenous information about the state. Our results have obvious implications for Blackwell-ranked changes in this exogenous information. Alternatively, instead of the additional information coming from an exogenous source or from other strategic agents, it may be gathered by the DM directly. Our results imply that the DM may be better off committing to not gather this information (e.g. by raising her own cost to do so) when the agent bears a disclosure cost.

From an applied point of view, it is interesting to consider alternative technologies of disclosure. We have assumed that the cost of disclosure/concealment is incurred only after an agent's signal is realized. In some settings, an agent may have to decide whether to incur a cost ex-ante: if he incurs the (positive) cost, then he can choose—at no further cost—whether to disclose or conceal any realized signal; if the cost is not incurred, the agent simply cannot disclose his signal. Such an alternative model could capture the cost for a lobbyist to buy access to a politician before he knows what issue he may be lobbying on, or for a firm to retain up front a consultant/expert to certify any information it may later want to make public. In such a setting, it is crucial whether the decision to incur the cost ex-ante is observable by the DM. If the decision is observable or overt (as is plausible in the lobbyist example), then it is straightforward that under linear preferences, the agent would never choose to incur the cost, no matter the behavior of any other agent. On the other hand, with an unobservable or covert decision (as is plausible in the firm example), notice that if an agent incurs the cost in equilibrium, behavior in the "subgame" is the same as in our baseline model with $c = 0$, regardless of other agents' behavior. It can be shown using IVP that more information disclosure by other agents reduces the ex-ante expected benefit for an agent from incurring the ex-ante cost: intuitively, because information validates the prior, the agent's expected payoff conditional on any signal that he will conceal goes down, whereas his payoff conditional on any signal that he will reveal does not change.

Throughout the current paper, we have taken the agents' information as exogenously given. In a companion paper (Kartik et al., 2014), we augment the current framework to study endogenous costly

information acquisition.[31] That paper shows that agents' information acquisition are strategic substitutes regardless of whether there are disclosure or concealment costs. Consequently, when information acquisition is endogenous, a DM can be worse off with more agents or when information acquisition becomes cheaper even when agents bear costs to conceal information.

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[31]Endogenous information acquisition followed by verifiable disclosure is studied in other frameworks by Matthews and Postlewaite (1985), Shavell (1994), Che and Kartik (2009), and Gentzkow and Kamenica (2012b).

A Proofs

A.1 Proofs for Section 3

Proof of Lemma 1. Partially differentiating Equation 1 with respect to the first argument yields

$$
\begin{aligned}
\frac{\partial \eta(\hat{s}, p, \pi)}{\partial \hat{s}} &= \frac{-p(1-p)f_\pi(\hat{s})}{(1-p+pF_\pi(\hat{s}))^2}(\pi - \mathbb{E}_\pi[s|s < \hat{s}]) + \frac{pF_\pi(\hat{s})}{1-p+pF_\pi(\hat{s})}\frac{f_\pi(\hat{s})}{F_\pi(\hat{s})}(\hat{s} - \mathbb{E}_\pi[s|s < \hat{s}]) \\
&= \frac{pf_\pi(\hat{s})}{1-p+pF_\pi(\hat{s})}\left(\frac{-(1-p)}{1-p+pF_\pi(\hat{s})}(\pi - \mathbb{E}_\pi[s|s < \hat{s}]) + (\hat{s} - \mathbb{E}_\pi[s|s < \hat{s}])\right) \\
&= \frac{pf_\pi(\hat{s})}{1-p+pF_\pi(\hat{s})}(\hat{s} - \eta(\hat{s}, p, \pi)).
\end{aligned}
$$

Hence, $\text{sign}[\frac{\partial \eta(\hat{s},p,\pi)}{\partial \hat{s}}] = \text{sign}[\hat{s} - \eta(\hat{s}, p, \pi)]$. Part (i) of the Lemma follows from the observation that for any p and π, $\eta(\underline{s}, p, \pi) = \eta(\overline{s}, p, \pi) = \pi$.

Partially differentiating with respect to the second argument and simplifying yields

$$
\frac{\partial \eta(\hat{s}, p, \pi)}{\partial p} = \frac{F_\pi(\hat{s})\left(\mathbb{E}_\pi[s|s < \hat{s}] - \pi\right)}{(1-p+pF_\pi(\hat{s}))^2},
$$

which proves the second part of the Lemma because $\mathbb{E}_\pi[s|s < \hat{s}] < \pi \iff \hat{s} < \overline{s}$ and $F_\pi(\hat{s}) > 0 \iff \hat{s} > \underline{s}$. $\square$

Proof of Proposition 1. The first part is straightforward and omitted. The second part follows from the fact that for any $p \in (0, 1)$ and π, $\eta(\cdot, p, \pi)$ is strictly decreasing on the domain $[0, \hat{s}]$, where $\hat{s}$ is the fixed point of $\eta(\cdot, p, \pi)$, which is interior (Lemma 1). The third part is because parameters can be chosen such that $c > 0$ and there are multiple solutions in s to $s - c = \eta(s, p_i, \pi)$, as depicted in Figure 1. This can be seen by fixing all parameters except c and p_i and then considering $p_i \to 1$ with a suitable choice of c; details are available on request. $\square$

Proof of Proposition 2. The first part of the Proposition is trivial and omitted. For the second part, fix any $p_i > \tilde{p}_i$ and let s^* and $\tilde{s}^*$ denote the corresponding highest equilibrium disclosure thresholds. Suppose, to contradiction, that $s^* > \tilde{s}^*$. First note that we must have $\tilde{s}^* \in (\underline{s}, \overline{s})$: clearly we cannot have $\tilde{s}^* = \overline{s}$, and if $\tilde{s}^* = \underline{s}$ then the fact that $\eta(\hat{s}, p, \pi)$ is weakly decreasing in p (Lemma 1) would imply that $s^* = \underline{s}$, a contradiction. Since $\tilde{s}^*$ is the highest equilibrium threshold at $\tilde{p}_i$, it follows that for any $s > \tilde{s}^*$, $\eta(s, \tilde{p}_i, \pi) < s - c$. But Lemma 1 then implies that for any $s > \tilde{s}^*$, $\eta(s, p_i, \pi) < s - c$, which implies that $s^* \leq \tilde{s}^*$, a contradiction. A similar argument can be used to establish the result for the lowest equilibrium thresholds. $\square$

A.2 Proofs for Subsection 3.2

Proof of Theorem 1. Recall the mapping $T(\cdot)$ defined in Equation 2. We claim that given any experiment, for any signal realization, say s, and posteriors $\beta_n(s)$ and $\beta_m(s)$, we have

$$\beta_n(s) = T(\beta_m(s), \overline{\beta}_n, \overline{\beta}_m) \equiv \frac{\beta_m \frac{\overline{\beta}_n}{\overline{\beta}_m}}{\beta_m \frac{\overline{\beta}_n}{\overline{\beta}_m} + (1 - \beta_m)\frac{1 - \overline{\beta}_n}{1 - \overline{\beta}_m}}. \tag{9}$$

as long as $\overline{\beta}_n, \overline{\beta}_m \in (0, 1)$.

To see this, consider for simplicity the case of either discrete signals or continuous signals and let $f(s|\omega)$ be the probability/density of signal s in state ω. Bayes rule in likelihood ratio form implies that for any individual $k \in \{m, n\}$, $\frac{\beta_k(s)}{1 - \beta_k(s)} = \frac{f(s|1)}{f(s|0)} \frac{\overline{\beta}_k}{1 - \overline{\beta}_k}$, and hence $\frac{\beta_n(s)}{1 - \beta_n(s)} = \frac{1 - \overline{\beta}_m}{\overline{\beta}_m} \frac{\overline{\beta}_n}{1 - \overline{\beta}_n} \frac{\beta_m(s)}{1 - \beta_m(s)}$. Manipulating this latter identity leads to (9).

Therefore, for any experiment $\mathcal{E}$, $\mathbb{E}^m_{\mathcal{E}}[\beta_n(\cdot)] = \mathbb{E}^m_{\mathcal{E}}[T(\beta_m(\cdot); \overline{\beta}_n, \overline{\beta}_m)]$. Differentiating $T(\cdot)$, we get

$$\frac{\partial T(\cdot)}{\partial \beta_m} = \frac{(\overline{\beta}_m - 1)\overline{\beta}_m(\overline{\beta}_n - 1)\overline{\beta}_n}{\left(\overline{\beta}_n \beta_m - \overline{\beta}_m(\overline{\beta}_n + \beta_m - 1)\right)^2},$$

$$\frac{\partial^2 T(\cdot)}{\partial(\beta_m)^2} = -\frac{2(\overline{\beta}_m - 1)\overline{\beta}_m(\overline{\beta}_n - 1)\overline{\beta}_n(\overline{\beta}_m - \overline{\beta}_n)}{\left(\overline{\beta}_m(\overline{\beta}_n - 1) - (\overline{\beta}_n - \overline{\beta}_m)\beta_m\right)^3} \propto \overline{\beta}_m - \overline{\beta}_n.$$

Therefore, $\frac{\partial^2 T(\cdot)}{\partial(\beta_m)^2} \geq (\text{resp. } \leq)0 \iff \overline{\beta}_m \geq (\text{resp. } \leq)\overline{\beta}_n$.

Part (i) of the result now follows from Blackwell's (1953) comparison-of-experiments theorem, which states that a garbling leads to a weak decrease (resp. increase) in the ex-ante expectation of any continuous and convex (resp. concave) function of an individual's posterior beliefs, no matter his prior. Part (ii) of the result is a straightforward consequence of combining part (i) with the observation that if $\mathcal{E}$ is totally uninformative (i.e. an individual's posterior is always the same as his prior for any signal realization, and hence $\mathcal{E}$ is a garbling of any other experiment) then $\mathbb{E}^m_{\mathcal{E}}[\beta_n(\cdot)] = \overline{\beta}_n$, whereas if $\mathcal{E}$ is perfectly informative (i.e. every signal perfectly reveals the state, and hence any other experiment is a garbling of $\mathcal{E}$) then $\mathbb{E}^m_{\mathcal{E}}[\beta_n(\cdot)] = \mathbb{E}^m_{\mathcal{E}}[\beta_m(\cdot)] = \overline{\beta}_m$. □

A.3 Proofs for Subsection 3.3

Proof of Lemma 2. Fix any equilibrium and any agent i and agent $j \neq i$. It suffices to show that the difference in the expected payoff for i from disclosing versus concealing is strictly increasing in s_i. Denote the expected payoff from concealing as $\mathbb{E}[\beta_{DM}(m_j, m_i = \phi)]$, where $\beta_{DM}(m_j, m_i = \phi)$ denotes the DM's equilibrium belief following any message m_j and nondisclosure by i, and the expectation is taken over m_j given i's beliefs under s_i. Because m_j is uncorrelated with s_i conditional on the state,

and i's belief about the state given s_i is just s_i,

$$\mathbb{E}[\beta_{DM}(m_j, m_i = \phi)] = s_i \mathbb{E}[\beta_{DM}(m_j, m_i = \phi)|\omega = 1] + (1 - s_i)\mathbb{E}[\beta_{DM}(m_j, m_i = \phi)|\omega = 0].$$

The derivative of the right hand side of the above equation with respect to s_i is strictly less than one because $\mathbb{E}[\beta_{DM}(m_j, m_i = \phi)|\omega = 1] < 1$, as beliefs lie in $[0, 1]$ and m_j cannot perfectly reveal the state. Since the expected payoff from disclosing is $s_i - c$, whose derivative with respect to s_i is one, we are done. $\qquad \square$

Proof of Lemma 3. Fix any p_j, p_i, $\hat{s}_i$, and s_i. $U(s_i, p_i, \hat{s}_j, p_j)$ is i's expectation of the DM's belief (viewed as random variable whose realization depends on j's message) under a prior s_i for i and $\eta(s_i, p_i, \pi)$ for the DM. It follows immediately from Theorem 1 that:

1. $s_i = \eta(s_i, p_i, \pi) \implies U(s_i, p_i, \hat{s}_j, p_j) = s_i$,

2. $s_i > \eta(s_i, p_i, \pi) \implies \eta(s_i, p_i, \pi) \leq U(s_i, p_i, \hat{s}_j, p_j) \leq s_i$,

3. $s_i < \eta(s_i, p_i, \pi) \implies \eta(s_i, p_i, \pi) \geq U(s_i, p_i, \hat{s}_j, p_j) \leq s_i$.

It is straightforward to see that because $p_j < 1$, the last inequalities in items 2 and 3 above are in fact strict, as an equality in either case requires j's message to be perfectly informative of the state. Finally, the other inequalities in items 2 and 3 are also strict if and only if $\hat{s}_j < \bar{s}$, as j's message is perfectly uninformative if and only if $\hat{s}_j = \bar{s}$. $\qquad \square$

Some of the proofs below deal with the best-response correspondence $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. To correctly account for boundary solutions, we generalize Equation 3 by formally defining $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ as follows: $s_i \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ if and only if (i) $U(s_i, p_i, \hat{s}_j, p_j) = s_i - c$, or (ii) $U(\underline{s}, p_i, \hat{s}_j, p_j) < \underline{s} - c$ and $s_i = \underline{s}$, or (iii) $U(\bar{s}, p_i, \hat{s}_j, p_j) > \bar{s} - c$ and $s_i = \bar{s}$.

Proof of Proposition 3. The transformation $T(\beta_m, \bar{\beta}_n, \bar{\beta}_m)$ defined in Equation 2 is increasing in $\bar{\beta}_n$. Lemma 1 shows that $\eta(\hat{s}_i, p_i, \pi)$ is decreasing in p_i. Hence, agent i's payoff from concealing signal s_i given a correct conjecture by the DM,

$$U(s_i, p_i, \hat{s}_j, p_j) = \mathbb{E}_{\hat{s}_j, p_j}[T(\beta(m_j; s_i), \eta(s_i, p_i, \pi), s_i)],$$

decreases in p_i for any signal s_i, while his payoff from disclosure, $s_i - c$, does not depend on p_i. Following the same argument as in the proof of Proposition 2, the largest and smallest best-response disclosure thresholds must decrease in p_i.

Let $\hat{s}_i^0$ be the smallest equilibrium threshold in the single-agent with i; recall that this is the unique equilibrium threshold if $c \leq 0$.

Consider first $c = 0$. It follows from Part 1 of Lemma 3 that agent i is indifferent between non-disclosure and disclosure when his signal is $\hat{s}^0$; hence, $\hat{s}^0 \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. Next, we claim there exists no other best-response disclosure threshold. Suppose, to contradiction, that $\hat{s}' > \hat{s}^0$ and $\hat{s}' \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. By Lemma 1, $\eta(\hat{s}', p_i, \pi) < \hat{s}'$. Lemma 3 then implies that $U(\hat{s}', p_i, \hat{s}_j, p_j) < \hat{s}'$. Therefore, agent i strictly prefers disclosure to non-disclosure when his signal is $\hat{s}'$, contradicting $\hat{s}' \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. A similar argument establishes that $\hat{s}' < \hat{s}^0 \implies s_i' \notin \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$.

Now consider $c < 0$. If $s_i > \hat{s}_i^0$, then $s_i - c > \eta(s_i, p_i, \pi)$. Since Lemma 3 implies that $U(s_i, p_i, \hat{s}_j, p_j) \leq \max\{s_i, \eta(s_i, p_i, \pi)\}$, it follows that $U(s_i, p_i, \hat{s}_j, p_j) < s_i - c$, and hence $s_i \notin \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. Furthermore, if $\hat{s}_j < \overline{s}$, then $\hat{s}_i^0 < \eta(\hat{s}_i^0, p_i, \pi)$ and Lemma 3 together imply $U(\hat{s}_i^0, p_i, \hat{s}_j, p_j) > \eta(s_i, p_i, \pi)$, and hence $\hat{s}_i^0 \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j) \iff \hat{s}_i^0 = \underline{s}$. Conversely, it is obvious that $\{s_i^0\} = \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$ if $\hat{s}_j = \overline{s}$. This proves part (i) of the result for $c < 0$. To prove part (ii), we first note from part (i) that if $s_i \in \hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$, $s_i \leq \hat{s}_i^0$ and hence $\eta(s_i, p_i, \pi) > s_i$. Theorem 1 then implies that any garbling of agent j's message increases $U(s_i, p_i, \hat{s}_j, p_j)$. Thus, an increase in $\hat{s}_j$ or a decrease in p_j—both of which represent a garbling of j's message—lowers agent i's non-disclosure payoff without affecting his disclosure payoff at signal s_i. Following the same argument as in the proof of Proposition 2, the largest and smallest best-response disclosure thresholds must increase.

We omit the proof for $c > 0$ as it follows a symmetric argument to that for $c < 0$; the only point to note is that here the definition of $\hat{s}_i^0$ as the smallest equilibrium threshold in the single-agent game is used to ensure that $s_i < \hat{s}_i^0 \implies s_i - c < \eta(s_i, p_i, \pi)$. $\qquad\square$

Proof of Proposition 4. Consider first the case $c \leq 0$. For each agent i, define the function

$$g_i(\hat{s}_i, \hat{s}_j; p_i, p_j) := \inf\{\hat{s}_i \mid U(\hat{s}_i, p_i, \hat{s}_j, p_j) \leq \hat{s}_i - c\}.$$

That is, $g_i(\cdot)$ gives the smallest element of $\hat{s}_i^{BR}(\hat{s}_j; p_i, p_j)$. Let $g = (g_i, g_j)$, and define

$$\mathbf{s}_*(p_i, p_j) := \inf\{(\hat{s}_i, \hat{s}_j) \mid g(\hat{s}_i, \hat{s}_j; p_i, p_j) \leq (\hat{s}_i, \hat{s}_j)\}.$$

By Proposition 3, $g(\cdot; p_i, p_j)$ is monotone increasing. Hence, $\mathbf{s}_*(p_i, p_j)$ is its smallest fixed point. It remains to be shown that $\mathbf{s}_*(p_i, p_j)$ is the smallest fixed point of the best-response correspondence $(\hat{s}_i^{BR}, \hat{s}_j^{BR})$. Let $\mathbf{s}$ be any other fixed point of the correspondence. Since g is monotone, we have $g(\mathbf{s}_* \wedge \mathbf{s}) \leq g(\mathbf{s}) \leq \mathbf{s}$ and $g(\mathbf{s}_* \wedge \mathbf{s}) \leq g(\mathbf{s}_*) \leq \mathbf{s}_*.[32]$ Thus, $g(\mathbf{s}_* \wedge \mathbf{s}) \leq \mathbf{s}_* \wedge \mathbf{s}$. By the definition of $\mathbf{s}_*$, this in turn implies that $\mathbf{s}_* \leq \mathbf{s}_* \wedge \mathbf{s}$, which is possible only if $\mathbf{s}_* \leq \mathbf{s}$, as required. Thus, this argument establishes that the smallest fixed point of the minimal best response is also the smallest fixed point among all best responses. In other words, $\mathbf{s}_*(p_i, p_j)$ is the *smallest* equilibrium and hence the best equilibrium for the DM. Proposition 3 establishes that g is decreasing in p_i. It follows from standard monotone comparative statics that the smallest fixed point of g decreases in p_i. It is also straightforward to see from the definition of g_i that g is increasing in c. Hence the best equilibrium increases in c.

---

[32]We follow the usual notational convention that $\mathbf{s} \wedge \mathbf{s}' \equiv (\min\{s_1, s_1'\}, \min\{s_2, s_2'\})$.

A parallel argument shows that the worst equilibrium also decreases in p_i and increases in c. Finally, a decrease in the equilibrium disclosure thresholds of both agents corresponds to an increase in informativeness of their messages. Hence, the DM's welfare improves whenever the equilibrium thresholds fall.

For the case $c > 0$, we keep the sign of $\hat{s}_i$ but flip the sign of $\hat{s}_j$ in the definition of g so that it is monotone in $(\hat{s}_i, -\hat{s}_j)$. The smallest fixed point of g then corresponds to the j-maximal equilibrium. By Proposition 3, a higher p_i decreases agent i's best response but increases agent j's best response. Hence, in a j-maximal equilibrium, $(\hat{s}_i, -\hat{s}_j)$ is decreasing in p_i. The same conclusion holds for an i-maximal equilibrium. Because a higher p_i lowers agent i's threshold but raises agent j's threshold, its effect on the informativeness of the agents' messages cannot be ranked in the sense of Blackwell (1953). Example 1 shows that the DM's welfare in the best equilibrium can strictly decrease when (i) p_i increases from 0 (which corresponds to the single agent case) to some positive level (which corresponds to the two agent case), or when (ii) c decreases from some positive value to a lower but still positive value. $\qquad\square$

Proof of Proposition 7. Write $V(\cdot)$ instead of $V_i(\cdot)$, denote $r \equiv \frac{1-\overline{\beta}_{DM}}{\overline{\beta}_{DM}} \frac{\overline{\beta}_i}{1-\overline{\beta}_i}$, and define

$$G(\beta, r) := V(T(\beta, r)) - V(\beta),$$

where $T(\beta_i, r) = \frac{\beta_i}{\beta_i + (1-\beta_i)r}$ is a shorthand for the $T(\beta, \overline{\beta}_{DM}, \overline{\beta}_i)$ defined in Equation 2. When $V(\cdot)$ is strictly monotone and $c = 0$, i's best response threshold must be such that $\mathbb{E}[G(\cdot, r)] = 0$ when r is determined by i's threshold type and the DM's nondisclosure belief.

Observe that when $r = 1$, then for any β, $T(\beta, 1) = \beta$ and hence $G(\beta, 1) = 0$. Furthermore, because $\frac{\partial T(\beta, r)}{\partial r} = -\frac{\beta(1-\beta)}{(\beta + (1-\beta)r)^2} < 0$ for all interior β, it follows that for any non-perfectly-informative experiment, $\mathbb{E}[G(\cdot, r)] = 0 \iff r = 1$. Thus, no matter j's disclosure strategy (so long as it is not perfectly informative of the state, which it cannot be since $p_j < 1$), i's best response threshold is such that $r = 1$, i.e. the DM's nondisclosure belief is the same as i's threshold type. But this is the same condition as in the single-agent game. $\qquad\square$

A.4 Proofs for Subsection 4.2

Proof of **??**. Fix any MLRP experiment $\mathcal{E} \equiv (Y, \mathcal{Y}, \{P_\omega\}_{\omega \in \Omega})$ and any two beliefs $\boldsymbol{\beta}' \geq_{LR} \boldsymbol{\beta}$. We want to show that

$$\rho(y', \boldsymbol{\beta}')\rho(y, \boldsymbol{\beta}) \geq \rho(y, \boldsymbol{\beta}')\rho(y', \boldsymbol{\beta}) \iff y' \geq y \tag{10}$$

Substituting in from **??** yields

$$\rho(y', \boldsymbol{\beta}')\rho(y, \boldsymbol{\beta}) \geq \rho(y, \boldsymbol{\beta}')\rho(y', \boldsymbol{\beta})$$

$$\iff \left(\sum_{\omega' \in \Omega} p(y'|\omega')\beta'(\omega')\right)\left(\sum_{\omega \in \Omega} p(y|\omega)\beta(\omega)\right) \geq \left(\sum_{\omega' \in \Omega} p(y|\omega')\beta'(\omega')\right)\left(\sum_{\omega \in \Omega} p(y'|\omega)\beta(\omega)\right)$$

$$\iff \sum_{\omega' \in \Omega}\sum_{\omega \in \Omega} \left[p(y'|\omega')p(y|\omega) - p(y'|\omega)p(y|\omega')\right]\beta'(\omega')\beta(\omega) \geq 0$$

$$\iff \sum_{k=0}^{L}\sum_{l=0}^{L} \left[p(y'|\omega_k)p(y|\omega_l) - p(y'|\omega_l)p(y|\omega_k)\right]\beta'(\omega_k)\beta(\omega_l) \geq 0$$

$$\iff \sum_{k=0}^{L}\sum_{l=k+1}^{L} \underbrace{\left[p(y'|\omega_k)p(y|\omega_l) - p(y'|\omega_l)p(y|\omega_k)\right]}_{A} \underbrace{\left[\beta'(\omega_k)\beta(\omega_l) - \beta'(\omega_l)\beta(\omega_k)\right]}_{B} \geq 0.$$

Observe that for any $l > k$, the term labeled B in the previous line is non-positive because $\boldsymbol{\beta}' \geq_{LR} \boldsymbol{\beta}$, and the term labeled A is non-positive (resp. non-negative) if $y' \geq y$ (resp. $y' \leq y$) because $\mathcal{E}$ is an MLRP-experiment. This proves (10). □

Proof of Theorem 2. **(To be written more clearly.)**

As defined in Equation 4, let $\mathcal{T}(\beta_m(\cdot); \overline{\beta}_n, \overline{\beta}_m)$ represent agent n's posterior mean about the state. Using the notation we developed earlier,

$$\mathcal{T}(\mathbf{x}) = \frac{\sum_k x_k r_k \omega_k}{\sum_k x_k r_k},$$

where $x_k = \beta_m(\omega_k|\cdot)$ and $r_k = \overline{\beta}_n(\omega_k)/\overline{\beta}_m(\omega_k)$.

We assume $\overline{\beta}_n \geq_{LR} \overline{\beta}_m$ throughout (with a symmetric argument applying if the inequality is reversed). Thus, r_k is an increasing sequence in k.

Suppose there is a signal $s \in \{s_1, \ldots, s_J\}$ that is informative about the state and satisfies MLRP. We write $x_k^j = \beta_m(\omega_k|s_j)$ and use the following notation:

$$N^j = \sum_k x_k^j r_k \omega_k;$$

$$D^j = \sum_k x_k^j r_k;$$

$$T^j = \frac{N^j}{D^j}.$$

By MLRP, D^j is an increasing sequence in j and T^j is also an increasing sequence in j.

For any non-negative weights c_j with $\sum_j c_j = 1$, we have

$$\frac{\sum_j c_j N^j}{\sum_j c_j D^j} - \sum_j c_j T^j = \sum_j c_j T^j \left(\frac{D^j}{\sum_j c_j D^j} - 1 \right).$$

Note that

$$\sum_j c_j \left(\frac{D^j}{\sum_j c_j D^j} - 1 \right) = 0,$$

and the expression in parentheses is single-crossing from below in j. Since T^j is an increasing sequence, this implies that

$$\frac{\sum_j c_j N^j}{\sum_j c_j D^j} \geq \sum_j c_j T^j.$$

Now, suppose there is another signal $\tilde{s} \in \{\tilde{s}_1, \ldots, \tilde{s}_I\}$ which is a garbling of s. By definition, this means that there exists non-negative b_{ij} with $\sum_i b_{ij} = 1$ such that, for any state ω_k,

$$\Pr[\tilde{s} = \tilde{s}_i | \omega_k] = \sum_j b_{ij} \Pr[s = s_j | \omega_k].$$

Let $\tilde{p}_i$ represent the unconditional probability of observing $\tilde{s} = \tilde{s}_i$ under agent m's prior belief $\overline{\beta}_m$. Likewise, let p_j represent the unconditional probability of observing $s = s_j$ under the same prior belief. If we multiply both sides of the equation above by $\beta_m(\omega_k)$ and divide by $\tilde{p}_i$, we obtain

$$\beta_m(\omega_k | \tilde{s} = \tilde{s}_i) = \sum_j b_{ij} \frac{p_j}{\tilde{p}_i} \beta_m(\omega_k | s = s_j).$$

Let $c_{ij} := b_{ij} p_j / \tilde{p}_i$, and let $\tilde{x}_k^i := \beta_m(\omega_k | \tilde{s} = \tilde{s}_i)$. Then we can write

$$\tilde{x}_k^i = \sum_j c_{ij} x_k^j.$$

Note that c_{ij} is non-negative. Moreover, for each i, we have

$$\sum_j c_{ij} = \frac{\sum_j b_{ij} p_j}{\tilde{p}_i} = \frac{\sum_j b_{ij} \sum_k \Pr[s = s_j | \omega_k] \beta_m(\omega_k)}{\tilde{p}_i} = \frac{\sum_k \Pr[\tilde{s} = \tilde{s}_i | \omega_k] \beta_m(\omega_k)}{\tilde{p}_i} = 1.$$

Agent m's expectation of agent n's posterior mean given information structure $\tilde{s}$ is:

$$
\begin{aligned}
\mathbb{E}_m[\mathcal{T}(\tilde{\mathbf{x}})] &= \sum_i \tilde{p}_i \frac{\sum_k \tilde{x}_k^i r_k \omega_k}{\sum_k \tilde{x}_k^i r_k} \\
&= \sum_i \tilde{p}_i \frac{\sum_j c_{ij} \sum_k x_k^j r_k \omega_k}{\sum_j c_{ij} \sum_k x_k^j r_k} \\
&= \sum_i \tilde{p}_i \frac{\sum_j c_{ij} N^j}{\sum_j c_{ij} D^j}.
\end{aligned}
$$

On the other hand, his expectation given information structure s is:

$$
\begin{aligned}
\mathbb{E}_m[\mathcal{T}(\mathbf{x})] &= \sum_j p_j \frac{\sum_k x_k^j r_k \omega_k}{\sum_k x_k^j r_k} \\
&= \sum_j \left(\sum_i b_{ij} \right) p_j \frac{\sum_k x_k^j r_k \omega_k}{\sum_k x_k^j r_k} \\
&= \sum_i \tilde{p}_i \sum_j c_{ij} \frac{N^j}{D^j},
\end{aligned}
$$

where the second equality uses $\sum_i b_{ij} = 1$ and the third equality uses the definition of c_{ij}.

Thus,

$$
\mathbb{E}_m[\mathcal{T}(\mathbf{x})] - \mathbb{E}_m[\mathcal{T}(\tilde{\mathbf{x}})] = \sum_i \tilde{p}_i \left(\sum_j c_{ij} T^j - \frac{\sum_j c_{ij} N^j}{\sum_j c_{ij} D^j} \right) \leq 0.
$$

$\square$

Proof of Lemma 4. For $s', s \in [\underline{s}, \bar{s}]$ and $p \in [0, 1]$, define

$$
K(s', s, p) := \frac{\sum_{\omega \in \Omega} f(s'|\omega) \pi(\omega)}{1 - p + p \sum_{\omega \in \Omega} F(s|\omega) \pi(\omega)}.
$$

Now fix any i, s_i and $s_i' \geq s_i$. From Equation 5 and Equation 6,

$$
\frac{\eta(\omega|s_i, p_i)}{\beta(\omega|s_i')} = \left(\frac{1 - p_i + p_i F(s_i|\omega)}{f(s_i'|\omega)} \right) K(s_i', s_i, p_i),
$$

which is weakly monotone in ω by the second part of Assumption 1. $\square$

Proof of Proposition 5. Fix any threshold equilibrium and pick any agent i with threshold s_i^*. First consider $c = 0$. If $\boldsymbol{\beta}(s_i^*) >_{LR} \boldsymbol{\eta}(s_i^*, p_i)$, then Theorem 2 (whose assumptions are verified by Lemma 4) implies that given signal s_i^*, agent i would strictly prefer disclosure, a contradiction. Analogously, it cannot hold that $\boldsymbol{\beta}(s_i^*) <_{LR} \boldsymbol{\eta}(s_i^*, p_i)$. By Lemma 4, $\boldsymbol{\eta}(s_i^*, p_i) = \boldsymbol{\beta}(s_i^*)$.

Now consider $c < 0$. If $\boldsymbol{\beta}(s_i^*) \geq_{LR} \boldsymbol{\eta}(s_i^*, p_i)$, then Theorem 2 implies that given signal s_i^*, agent i would strictly prefer disclosure, a contradiction. By Lemma 4, $\boldsymbol{\eta}(s_i^*, p_i) >_{LR} \boldsymbol{\beta}(s_i^*)$.

The case of $c > 0$ is analogous to $c < 0$. $\qquad\qquad\square$

B Supplementary Appendix (Not For Publication)

B.1 Discussion of Theorem 2

This section proves the claims in Remark 1 by showing that the conclusion of Theorem 2 can fail with a non-MLRP-experiment (Example 3) or if the priors are not likelihood-ratio ordered (Example 4).

Example 3. *Let $\Theta = \{0, 1, 2\}$. Consider two individuals, m and n, with priors $\overline{\beta}_m$ and $\overline{\beta}_n$ satisfying:*

$$\frac{\overline{\beta}_m(0)}{\overline{\beta}_n(0)} < \frac{\overline{\beta}_m(1)}{\overline{\beta}_n(1)} = 2 < \frac{\overline{\beta}_m(2)}{\overline{\beta}_n(2)}.$$

Plainly, $\overline{\beta}_m >_{LR} \overline{\beta}_n$ and hence $M(\overline{\beta}_m) > M(\overline{\beta}_n)$, where recall that $M(\cdot)$ denotes the expectation operator.

   Consider an experiment $\mathcal{E}$ with a binary signal space $\{l, h\}$ and

$$\Pr(l|1) = 1 > \Pr(l|0) = \Pr(l|2) = 0,$$
$$\Pr(h|1) = 0 < \Pr(h|0) = \Pr(h|2) = 1.$$

Thus, signal l reveals state 1 whereas signal h just reveals that the state is not 1, but no relative information about states 0 and 2. Plainly, this not an MLRP-experiment. It is straightforward to compute that

$$\mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))] = 2\beta_n(1) + (1 - 2\beta_n(1))\frac{\beta_n(2)}{\beta_n(0) + \beta_n(2)}.$$

On the other hand, $M(\overline{\beta}_n) = \overline{\beta}_n(1) + 2\overline{\beta}_n(2)$. So $M(\overline{\beta}_m) > M(\overline{\beta}_n) > \mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))]$ if and only if

$$2\overline{\beta}_n(1) + (1 - 2\overline{\beta}_n(1))\frac{\overline{\beta}_n(2)}{\overline{\beta}_n(0) + \overline{\beta}_n(2)} < \overline{\beta}_n(1) + 2\overline{\beta}_n(2),$$

which clearly has solutions, for example $\overline{\beta}_n(0) = 1/8$, $\overline{\beta}_n(1) = 1/2$, and $\overline{\beta}_n(2) = 3/8$.

Example 4. *Let $\Theta = \{0, 1, 2\}$. Now consider an experiment $\mathcal{E}$ with binary signals $\{l, h\}$ that satisfies MLRP:*

$$\frac{\Pr(h|0)}{\Pr(l|0)} < \frac{\Pr(h|1)}{\Pr(l|1)} < \frac{\Pr(h|2)}{\Pr(l|2)}.$$

Let m and n be individuals with priors $\overline{\beta}_m$ and $\overline{\beta}_n$ such that $\overline{\beta}_m(1) = 1$, $\overline{\beta}_n(0) = \overline{\beta}_n(2) = \frac{1}{2}$. Plainly, these priors are not likelihood-ratio ordered and $M(\overline{\beta}_m) = M(\overline{\beta}_n) = 1$. Observe that

$$\mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))] = \Pr(l|1)M(\boldsymbol{\beta}_n(l)) + \Pr(h|1)M(\boldsymbol{\beta}_n(h)),$$

where by construction $\beta_n(2|l) < 1/2 < \beta_n(2|h)$ and $\beta_n(1|l) = \beta_n(1|h) = 0$. It follows that if $\Pr(h|1) < \Pr(l|1)$, then $\mathbb{E}_{\mathcal{E}}^m[M(\boldsymbol{\beta}_n(\cdot))] < 1 = M(\overline{\beta}_n) = M(\overline{\beta}_m)$.

B.2 Discussion of Assumption 1

This section elaborates on the second part of of Assumption 1. Assume Part 1 of Assumption 1 holds, viz. strict MLRP. For any $s, s' \in [\underline{s}, \overline{s}]$ and $p \in [0, 1]$ denote

$$L(\omega, p, s, s') := \frac{1 - p + pF(s|\omega)}{f(s'|\omega)}.$$

Taking $p = p_i$, Part 2 of Assumption 1 is equivalent to:

$$\text{for any } s \leq s', L(\omega, p, s, s') \text{ is weakly monotone in } \omega. \tag{11}$$

Since $L(\omega, p, \underline{s}, s') = \frac{1-p}{f(s'|\omega)}$, (11) cannot hold unless

$$\text{for any } s', f(s'|\omega) \text{ is weakly monotone in } \omega. \tag{12}$$

Because of strict MLRP, (12) is in fact equivalent to the existence of an uninformative signal, i.e.

$$\exists s^u \in (\underline{s}, \overline{s}) : \forall \omega', \omega, f(s|\omega') = f(s|\omega), \tag{13}$$

which, because of strict MLRP again, is further equivalent to

$$\exists s^u \in (\underline{s}, \overline{s}) : \forall \omega' > \omega, \ s > (=, <)s^u \implies f(s|\omega') > (=, <)f(s|\omega).^{33} \tag{14}$$

So assume (14).

- Observe that for any $s' \geq s^u$, $L(\cdot, p, s, s')$ is weakly decreasing for any p and s because MLRP implies that $F(s|\cdot)$ is decreasing (first order stochastic dominance, FOSD) and $f(s'|\cdot)$ is weakly increasing.

- For any $s' \leq s^u$ and $\omega' > \omega$, let

$$D(\omega, \omega', p, s, s') := L(\omega', p, s, s') - L(\omega, p, s, s') = \frac{1 - p + pF(s|\omega')}{f(s'|\omega')} - \frac{1 - p + pF(s|\omega)}{f(s'|\omega)}.$$

Differentiating yields

$$\frac{\partial D(\cdot)}{\partial p} = \frac{1 - F(s|\omega)}{f(s'|\omega)} - \frac{1 - F(s|\omega')}{f(s'|\omega')} \leq 0 \text{ with strict inequality if } s' < s^u, \tag{15}$$

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[33]To see that (12) implies (13), note first that it cannot be that $f(\cdot|\omega_L) = f(\cdot|\omega_1)$ because of strict MLRP. By continuity and the fact that these are densities, there must be some s^u such that $f(s^u|\omega_L) = f(s^u|\omega_1)$. By (12), $f(s^u|\omega)$ is constant in ω. By strict MLRP again, there cannot be any $s \neq s^u$ and $\omega' \neq \omega$ such that $f(s|\omega) = f(s|\omega')$. Consequently, (12) implies for any $s \neq s^u$, $f(s|\cdot)$ is either strictly increasing or strictly decreasing, which further implies that $s^u \in (\underline{s}, \overline{s})$. That (13) implies (14) is because strict MLRP requires that for any $s < (>)s^u$, $f(s^u|\omega)/f(s|\omega)$ must be strictly increasing (decreasing) in ω.

where the inequality is because $f(s'|\cdot)$ is non-increasing given $s \leq s^u$ (and strictly decreasing if $s < s^u$) and FOSD. For any $s' \leq s^u$, and any s, ω, ω', let

$$p^*(\omega, \omega', s, s') := \max\{p \in [0,1] : D(\omega, \omega', p, s, s') \geq 0\}.^{34}$$

Note that this is is well-defined because $s' \leq s^u$ implies $D(\omega, \omega', 0, s, s') \geq 0$; furthermore, $p^*(\cdot)$ is continuous in s and s' and (15) implies that

$$\text{sign}[D(\omega, \omega', p, s, s')] = \text{sign}[p^*(\omega, \omega', s, s') - p]. \tag{17}$$

We also define for any $s' \leq s^u$ and s,

$$\underline{p}(s, s') := \min_{\omega, \omega'} p^*(\omega, \omega', s, s'),$$

$$\overline{p}(s, s') := \max_{\omega, \omega'} p^*(\omega, \omega', s, s').$$

It follows from Equation 17 that for any $s \leq s^u$ and any s, $L(\omega, p, s, s')$ is monotone in ω if and only if $p \notin (\underline{p}(s, s'), \overline{p}(s, s')).^{35}$

Putting the two bullet points above together: under (12), (11) holds if and only if

$$p \notin P := \bigcup_{s \leq s' \leq s^u} (\underline{p}(s, s'), \overline{p}(s, s')).$$

Thus, in general, (11) (and hence Assumption 1) may hold for some value of p but not others. However, if (11) is to hold for all $p \in [0,1]$, then we must have $P = \emptyset$, which is equivalent to the requirement that

$$\forall s \leq s' \leq s^u : \underline{p}(s, s') = \overline{p}(s, s'),$$

or still equivalently,

$$\forall s \leq s' \leq s^u, \exists p^*(s, s') \text{ such that } \forall \omega < \omega' : p^*(\omega, \omega', s, s') = p^*(s, s'). \tag{18}$$

In words, in the relevant region, the critical value $p^*(\omega, \omega', s, s')$ cannot depend on ω, ω'.

Example 2 continued. The densities from Equation 7 satisfy the strict MLRP because for any $s' > s$

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[34]From the formula for $D(\cdot)$, we have

$$p^*(\omega, \omega', s, s') = \min\left\{1, \frac{f(s'|\omega) - f(s'|\omega')}{f(s'|\omega) - f(s'|\omega') + F(s|\omega)f(s'|\omega') - F(s|\omega')f(s'|\omega)}\right\}. \tag{16}$$

[35]Note that we may have $(\underline{p}(s, s'), \overline{p}(s, s')) = \emptyset$; in particular, this is the case when $s' = s^u$ because for any s, ω, ω', $p^*(\omega, \omega', s, s^u) = 0$ because $D(\omega, \omega', 0, s, s^u) = 0$.

and $\omega' > \omega$,

$$\frac{f(s'|\omega')}{f(s|\omega')} - \frac{f(s'|\omega)}{f(s|\omega)} = \frac{(s'-s)\left(h(\omega)g\left(\omega'\right) - g(\omega)h\left(\omega'\right)\right)}{(sg(\omega) + (1-s)h(\omega))\left(sg\left(\omega'\right) + (1-s)h\left(\omega'\right)\right)} > 0,$$

where the inequality is because $\frac{g(\omega')}{h(\omega')} > \frac{g(\omega)}{h(\omega)}$ by hypothesis. Substituting into Equation 16 from Equation 7, we compute that in this example,

$$p^*(\omega, \omega', s, s') = \min\left\{1, \frac{2s'-1}{(s-1)(s-2s'+1)}\right\},$$

which is independent of ω, ω'. Hence, (18) is satisfied, which verifies Assumption 1 for any p_i.

B.3 Perfectly correlated signals

Proposition 8. *Assume perfectly correlated signals, $c = 0$, and that the two agents are opposite biased. Then, each agent discloses strictly less than what he does in his single-agent game.*

Proof of Proposition 8. We prove it for the upward biased agent; the argument is symmetric for the other agent. Without loss, let agent 1 be upward biased and agent 2 be downward biased. Let s_1^* denote the single-agent threshold, i.e. $\eta(s_1^*) = s_1^*$. Write $\eta(\hat{s}_1, \hat{s}_2)$ as the non-disclosure belief (in the event both agents do not disclose) in the two-agent game when the respective thresholds are $\hat{s}_1$ and $\hat{s}_2$. Even though the DM's updating is not separable as in our baseline model, it is clear that $\eta(\hat{s}_1, \hat{s}_2) \geq \eta(\hat{s}_1)$, with equality if and only if $\hat{s}_2 = \underline{s}$. This follows from the simple observation that the non-disclosure event can be viewed as the union of two events: (i) $m_1 = m_2 = \phi$ and $s_2 = \phi$; and (ii) $m_1 = m_2 = \phi$ and $s_2 > \hat{s}_2$. Conditional on the first event, the DM's posterior is just $\eta(\hat{s}_1)$, whereas conditional on the second event, the posterior is larger than $\eta(\hat{s}_1)$ (strictly if and only if $\hat{s}_2 > \underline{s}$). It follows that for any $\hat{s}_2 > \underline{s}$, because $\eta(\hat{s}_1) \geq \hat{s}_1$ for all $\hat{s}_1 \leq s_1^*$, if the DM conjectures thresholds $(\hat{s}_1, \hat{s}_2)$, agent 1 with signal $s_1 = \hat{s}_1$ will strictly prefer nondisclosure to disclosure, where we are using the fact that if agent 2 discloses, he necessarily discloses s_1. Therefore, since agent 2 will use a threshold strictly larger than $\underline{s}$ in any equilibrium, any equilibrium involves agent 1's threshold being strictly larger than s_1^*. $\square$

B.4 Non-linear utility functions

As discussed in Section 4 of the main text, if the agent has a utility function $V_i(\beta_{DM})$, then how he responds to changes in the other agent's disclosure depends on properties of $\mathbb{E}[V_i(T(\beta_i, \overline{\beta}_i, \overline{\beta}_{DM})) - V_i(\beta_i)]$. Below, we show through a family of exponential utility functions how our conclusions are affected by departures from linearity of $V_i(\cdot)$. To this end, consider the more succinct representation:

$\mathbb{E}[G(\beta, r)]$, where $G : [0,1] \times \mathbb{R}_+ \to \mathbb{R}$ is defined as

$$G(\beta, r) := V(T(\beta, r)) - V(\beta), \tag{19}$$

where $T(\beta, r) = \frac{\beta}{\beta + (1-\beta)r}$. Here, r is a shorthand for $\frac{1-\bar{\beta}_{DM}}{\bar{\beta}_{DM}} \frac{\bar{\beta}_i}{1-\bar{\beta}_i}$; note that $r > 1 \iff \beta_i > \beta_{DM}$ and $r < 1 \iff \beta_i < \beta_{DM}$. Thus, under a disclosure cost $(c > 0)$ the relevant case is $r > 1$ if the agent is upward biased and $r < 1$ if the agent is downward biased; under a concealment cost $(c < 0)$ the relevant case is $r < 1$ if the agent is upward biased and $r > 1$ if the agent is downward biased.

In the following proposition, we say that an upward biased agent i's disclosure is a strategic substitute (resp. complement) to j's if whenever j's message is more Blackwell-informative, i's largest and smallest best response disclosure thresholds increase (resp. decrease).

Proposition 9. *Assume $V_i(\beta) = \gamma \beta^\alpha$, where either $\gamma, \alpha > 0$ or $\gamma, \alpha < 0$, so that agent i is upward biased. Then, i's disclosure is:*

1. *a strategic substitute to j's under disclosure cost if $0 < \alpha \leq 1$ and $\gamma > 0$.*

2. *a strategic substitute to j's under concealment cost if $\alpha < 0$ and $\gamma < 0$.*

3. *a strategic complement to j's under disclosure cost if $\alpha \leq -1$ and $\gamma < 0$.*

Part 1 of Proposition 9 is a generalization of Part 2 of Proposition 3 to some non-linear preferences; on the other hand, Parts 2 and 3 of Proposition 9 show how our main findings of strategic complementary under concealment cost and strategic substitutability under disclosure cost can actually be reversed for other non-linear preferences.[36]

Given the discussion preceding Proposition 9, and invoking Blackwell's results as in the proof of Theorem 1, Proposition 9 is a straightforward consequence of the following lemma.

Lemma 5. *If $V(\beta) = \beta^\alpha$ then $G(\beta, r)$ defined in (19) is:*

1. *convex in β if $0 < \alpha \leq 1$ and $r > 1$;*

2. *concave in β if $\alpha < 0$ and $r < 1$;*

3. *convex in β if $\alpha < -1$ and $r > 1$;*

Proof of Lemma 5. Denoting partial derivatives with subscripts as usual, we compute $T_\beta(\cdot) = \frac{r}{(\beta + (1-\beta)r)^2}$, $T_{\beta\beta}(\cdot) = \frac{2r(r-1)}{(\beta + (1-\beta)r)^3}$, and hence

$$G_{\beta\beta}(\cdot) = V''\left(\frac{\beta}{\beta + (1-\beta)r}\right)\left(\frac{r^2}{(\beta + (1-\beta)r)^4}\right) + V'\left(\frac{\beta}{\beta + (1-\beta)r}\right)\left(\frac{2r(r-1)}{(\beta + (1-\beta)r)^3}\right) - V''(\beta).$$

---

[36]Note that one has to be careful with the analog of Proposition 9 for the case of a downward biased agent, because the direction of disagreement between i and the DM reverses, as highlighted before the proposition. Thus, if $V_i(\beta) = -\gamma\beta^\alpha$, then in each part of Proposition 9 one should replace "disclosure cost" with "concealment cost" and vice-versa.

Plugging in $V_i(\beta) = \beta^\alpha$ and doing some algebra yields

$$G_{\beta\beta}(\cdot) = (\beta(\beta + (1-\beta)r))^{-2}\, \alpha \left(r \left(\frac{\beta}{\beta + (1-\beta)r} \right)^\alpha (r(\alpha + 2\beta - 1) - 2\beta) - (\alpha - 1)(\beta + (1-\beta)r)^2 \beta^\alpha \right)$$

$$\propto \alpha \left[(1-\alpha) + \frac{r(2\beta(r-1) - r(1-\alpha))}{(\beta + (1-\beta)r)^{\alpha+2}} \right] =: H(\beta, \alpha, r).$$

Observe that $H(0, \alpha, r) = \alpha(1-\alpha)(1 - r^{-\alpha})$, and hence if $\alpha < 1$ and $\alpha \neq 0$ then $\mathrm{sign}[H(0, \alpha, r)] = \mathrm{sign}[r-1]$. Differentiating yields

$$H_\beta(\cdot) = \frac{\alpha(\alpha + 1)(r-1)r(\alpha r + 2\beta(r-1))}{(\beta + (1-\beta)r)^{\alpha+3}}. \tag{20}$$

We now consider four cases:

1. Suppose $0 < \alpha \leq 1$ and $r > 1$. Then $H(0, \alpha, r) \geq 0$ and $H_\beta(\cdot) > 0$, and hence $H(\beta, \alpha, r) > 0$ for all $\beta \in (0, 1)$.

2. Suppose $-1 \leq \alpha < 0$ and $0 \leq r < 1$. Then $H(0, \alpha, r) < 0$ and $H_\beta(\cdot) \leq 0$, and hence $H(\beta, \alpha, r) < 0$ for all $\beta \in (0, 1)$.

3. Suppose $\alpha < -1$ and $r > 1$. Then $H(0, \alpha, r) > 0$ and $H(1, \alpha, r) = \alpha(r-1)(\alpha - 1 + r(1+\alpha)) > 0$. We will show that $H_\beta(\beta, \alpha, r) = 0 \implies H(\beta, \alpha, r) > 0$, which combines with the previous two inequalities to imply that $H(\cdot) > 0$. Accordingly, assume $H_\beta(\beta, \alpha, r) = 0$. Then, from Equation 20, $\beta = \frac{\alpha r}{2(1-r)}$, which because $\beta \leq 1$ and $\alpha < -1$ implies $\alpha \in (-2, -1)$ and $r \geq \frac{2}{2+\alpha}$. Furthermore,

$$H \left(\frac{\alpha r}{2(1-r)}, \alpha, r \right) = \alpha \left[(1-\alpha) + \frac{r(-\alpha r - r(1-\alpha))}{\left(\frac{\alpha r}{2(1-r)} + \left(1 - \frac{\alpha r}{2(1-r)} \right) r \right)^{\alpha+2}} \right]$$

$$= \alpha \left[1 - \alpha - r^2 \left(\frac{2}{r(\alpha + 2)} \right)^{\alpha+2} \right].$$

The derivative of the above expression with respect to r is $\alpha^2 \left(\frac{2}{\alpha+2} \right)^{\alpha+2} r^{-\alpha-1}$, which is strictly positive given $\alpha \in (-2, -1)$ and $r > \frac{2}{\alpha+2} > 2$. Moreover, when evaluated with $r = 2$, the expression reduces to $\alpha \left(1 - \alpha - \frac{4}{(\alpha+2)^2} \right)$, which is strictly positive given $\alpha \in (-2, -1)$. Therefore, $H \left(\frac{\alpha r}{2(1-r)}, \alpha, r \right) > 0$, as was to be shown.

4. Suppose $\alpha < -1$ and $0 \leq r < 1$. Then $H(0, \alpha, r) < 0$ and $H(1, \alpha, r) = \alpha(r-1)(\alpha - 1 + r(1+\alpha)) < 0$. As argued in the previous case, $H_\beta(\beta, \alpha, r) = 0$ requires $\alpha \in (-2, -1)$ and $r \geq \frac{2}{2+\alpha} > 2$, which is not possible given that we have assumed $r < 1$. Thus, $H_\beta(\cdot)$ has a constant sign in the relevant domain, which implies that $H(\cdot) < 0$ in the relevant domain. $\square$

B.5 Welfare examples under disclosure cost

This section provides the calculations that verify the claims made in Example 1.

Preliminary: by Lemma 2 of the paper, any equilibrium is a threshold equilibrium, i.e., both agents use threshold strategies. For an upward biased agent i, if $s_i \in \{0, 1-\gamma, \gamma, 1\}$, then it means respectively that the agent plays a pure strategy of concealing one signal $\underline{s}$ only, concealing two signals up to signal s^l, concealing three signals up to signal s^h, and concealing four signals up to signal $\overline{s}$. We allow mixed strategies. For example, the agent can choose to conceal signals $\underline{s}$ and s^l, and randomize on concealing and disclosing signal s^h with some probability.

An agent's payoff of disclosing his first, second, third and fourth signal are respectively $0 - c$, $1 - \gamma - c$, $\gamma - c$ and $1 - c$. Note that $\gamma > \pi$ implies that an agent at least conceals the first two signals. That is $s_i \geq 1 - \gamma$ for an upward biased agent i.

Parameter Case A: $\gamma = 0.7$, $\delta = 0.7$, $p_1 = p_2 = 0.8 \equiv p$, $c = 0.36$

Then disclosing the third signal gives an agent $\gamma - c = 0.34$ and disclosing the fourth signal gives $1 - c = 0.64$.

Claim A1. With a single upward biased agent, the best equilibrium has the agent only disclosing signals s^h and $\overline{s}$ (and symmetrically if the agent is downward biased).

Proof. WLOG, we prove for a single agent who is upward biased. Since an agent at least conceals two signals, it suffices to show that there is an equilibrium with two signals hidden. That is, given that the agent is believed to conceal two signals, $\hat{s} = 1 - \gamma$, the payoff of concealing any signal is:

$$\eta(1 - \gamma, p, \frac{1}{2}) = \frac{1 - p + p(1 - \gamma)\delta}{2 - p} = 0.306667 < \gamma - c.$$

Therefore, the agent chooses to disclose the third signal. □

Claim A2. With opposite biased agents, the best equilibrium has the upward biased agent only disclosing signal $\overline{s}$ and the downward biased agent only disclosing signal $\underline{s}$. The DM's welfare in this equilibrium is strictly lower than in the best equilibrium with either agent alone.

Proof. Suppose agent 1 is upward biased and agent 2 is downward biased.

Step 1. That is, there is no equilibrium where one agent conceals all four signals and the other agent conceals two or more than two signals.

First, there does not exist an equilibrium where agent 1 conceals four signals and agent 2 conceals two signals. Suppose there is. Then given that agent 2 conceals two signals and agent 1 is believed to

conceal four signals, the payoff to agent 1 of concealing signal $\bar{s}$ is:

$$\delta(1-\gamma)p(1-\gamma) + (1-\delta(1-\gamma)p)\frac{1-p+p(\delta\gamma+1-\delta)}{2-p} = 0.627253 < 1-c.$$

With probability $\delta(1-\gamma)p$ DM receives s^l from agent 2 and reaches a posterior of $1-\gamma$. With the complementary probability DM receives no disclosure from any agent and reaches a posterior of $\frac{1-p+p(\delta\gamma+1-\delta)}{2-p}$. Therefore, agent 1 wants to disclose $\bar{s}$, which forms a contradiction.

By strategic substitution, if agent 2 conceals more than two signals and agent 1 is believed to conceal four signals, agent 1 wants to disclose $\bar{s}$ as well. That is, there is no equilibrium where one agent conceals all four signals and the other agent conceals two or more than two signals.

Step 2. There is an equilibrium where both agents conceal three signals. Given that agent 2 conceals three signals and agent 1 is believed to conceal three signals, agent 1's payoff of concealing s^h is:

$$(1-p(1-\gamma)(1-\delta))\frac{1}{2} = 0.464 > \gamma - c$$

If agent 2 discloses a signal, then it must be $\underline{s}$, which gives agent 1 a payoff of 0. When agent 2 conceals a signal, given the symmetric setup, DM's posterior is $\frac{1}{2}$. Since the payoff of concealing s^h is greater than the payoff of disclosing it, agent 1 conceals s^h.

Step 3. There is no equilibrium where one agent conceals three signals and the other conceals less than three signals. First we claim that there is no equilibrium where agent 1 conceals two signals and agent 2 conceals three signals. Given that agent 2 conceals three signals and agent 1 is believed to conceal two signals, agent 1's payoff of concealing s^h is:

$$(1-(1-\gamma)(1-\delta)p)\frac{\eta(1-\gamma,p,\frac{1}{2})}{\eta(1-\gamma,p,\frac{1}{2}) + (1-\eta(1-\gamma,p,\frac{1}{2}))(1-p+p\delta)} = 0.341395 > \gamma - c.$$

Therefore, agent 1 conceals s^h, which rules out the existence of such an equilibrium. This further implies that there is no equilibrium where both agents conceal two signals.

Now suppose that agent 1 conceals two signals and randomizes on concealing s^h with probability $\lambda \in (0,1)$. Note that $\eta(\hat{s}_1, p, \frac{1}{2}) = \frac{1-p+p(1-\gamma)\delta+p\gamma\delta\lambda}{2-p+p\delta\lambda}$ is increasing in λ for the parameter case we consider. The payoff of concealing s^h is better than when he is believed to conceal only two ($\lambda = 0$). This implies that agent 1 also strictly prefers to conceal s^h.

Step 4. There are no equilibria where one agent conceals two signals and mixes on concealing the third while the other agent conceals three signals and mixes on concealing the fourth. WLOG, suppose agent 2 conceals two signals and mixes on the third in equilibrium and agent 1 conceals three signals and mixes on the fourth in equilibrium. Recall that, from Step 2, given agent 2 conceals two signals and agent 1 is believed to conceal four signals, agent 1 wants to disclose the fourth signal. Then, given that agent 2 conceals more than two signals and agent 1 is still believed to conceal four signals, strategic

substitution implies that agent 1 wants to disclose the fourth signal as well. Now instead suppose that agent 1 is believed to conceal three signals and randomize on the fourth signal, i.e., less than four signals. Since $\eta(\hat{s}_1, p, \frac{1}{2})$ is strictly increasing over $\hat{s}_1 \geq \gamma$ and reaches its maximum at $\eta(1, p, \frac{1}{2}) = \frac{1}{2}$, agent 1's payoff of concealing the fourth signal is even worse than if agent 1 is believed to conceal four signals, so agent 1 wants to disclose the fourth signal. Therefore, the above proposed mixed strategy equilibrium does not exist.

Step 5. From Step 3 and Step 4, one learns that the best equilibrium for DM must be that both agents conceal three signals. In this equilibrium, the expected payoff of the DM is:

$$-(1 - p(1 - \delta))\frac{1}{4} = -0.19$$

In the best equilibrium under a single agent, i.e., the agent conceals two signals up to s^l, the expected loss of the DM is:

$$-(1 - \frac{1}{2}p)\eta(1 - \gamma, p, \frac{1}{2})(1 - \eta(1 - \gamma, p, \frac{1}{2})) + \frac{1}{2}p\delta\gamma(1 - \gamma) = -0.186373$$

Therefore, comparing the best equilibrium, the DM is worse off with two opposing agents than with only a single agent. □

Parameter Case B: $\gamma = 0.7$, $\delta = 0.7$, $p_1 = p_2 = 0.8 \equiv p$, $c = 0.38$

Then disclosing the third signal gives an agent $\gamma - c = 0.32$ and disclosing the fourth signal gives $1 - c = 0.62$. Since the difference from Parameter Case A is only in the disclosure cost, the payoff of concealing a particular signal in a particular equilibrium is the same as the corresponding payoff under Parameter Case A.

Claim B. With opposite biased agents, the best equilibrium gives the DM a payoff equal to or higher than an equilibrium where the upward biased agent discloses signal s^h and $\bar{s}$ and the downward biased agent discloses no signal. Contrasting Parameter Case B with Parameter Case A, a higher cost c gives DM a better payoff.

Proof. Step 1. There exists an equilibrium where agent 1 conceals two signals and agent 2 conceals four signals. Given agent 2 conceals all signals, agent 1's payoff of concealing a signal is $\eta(\gamma, p, \frac{1}{2}) = 0.306667 < \gamma - c$, so agent 1 conceals only two signals. Given agent 1 conceals two signals, agent 2's payoff of concealing the fourth signal ($\underline{s}$) is $0.627253 > 1 - c$, so agent 2 will conceal all of his signals.

There may also exist an equilibrium where agent 2 conceals four signals and agent 1 conceals more than two signals, but this equilibrium is dominated by the above one in terms of DM's welfare.

Step 2. There is also an equilibrium where both conceal three signals because the payoff of concealing the third signal $0.464 > \gamma - c$.

Step 3. There is no equilibrium where one conceals three signals and the other conceals less than three signals because $0.341395 > \gamma - c$. This further implies that there is no equilibrium where both agents conceal two signals.

Step 4. There may exist a mixed strategy equilibrium where one agent conceals two signals and mixes on concealing the third while the other agent conceals three signals and mixes on concealing the fourth. This equilibrium's payoff may dominate the equilibrium in Step 1.

Step 5. Step 5 of Claim A2 already shows that the equilibrium in Step 1 here is better for DM than that in Step 2. Therefore, comparing the best equilibrium of the two parameter cases, the DM is better off when c is higher. □

Parameter Case C: $\gamma = 0.8$, $\delta = 0.7$, $p_1 = p_2 = 0.4 \equiv p$, $c = 0.385$

Then disclosing the third signal gives an agent $\gamma - c = 0.415$ and disclosing the fourth signal gives $1 - c = 0.615$.

Claim C1. With a single upward biased agent, the best equilibrium has the agent only disclosing signals s^h and $\bar{s}$ (and symmetrically if the agent is downward biased).

Proof is the same as that of Claim A1, adapted to the current parameter case.

Claim C2. With opposite biased agents, the best equilibrium has the upward biased agent only disclosing signal $\bar{s}$ and the downward biased agent only disclosing signal $\underline{s}$. The DM's welfare in this equilibrium is strictly lower than in the best equilibrium with either agent alone.

Proof is the same as that of Claim A2, adapted to the current parameter case.

Claim C3. With two upward biased agents, the best equilibrium has each agent only disclosing signal $\bar{s}$. The DM's welfare in this equilibrium is strictly lower than in the above single-agent equilibrium.

Proof. Step 1. That is, there is no equilibrium where one agent conceals all four signals and the other agent conceals two or more than two signals.

First, there does not exist an equilibrium where agent 1 conceals four signals and agent 2 conceals two signals. Suppose there is. Then given that agent 2 conceals two signals and agent 1 is believed to conceal four signals, the payoff to agent 1 of concealing signal $\bar{s}$ is:

$$\delta \gamma p \gamma + (1 - \delta)p + (1 - \delta \gamma p - (1 - \delta)p) = 0.56816 < 1 - c.$$

With probability $\delta \gamma p$ DM receives s^h from agent 2 and reaches a posterior of γ. With probability $(1-\delta)p$ DM receives $\bar{s}$ from agent 2 and reaches a posterior of 1. With the remaining probability DM receives no disclosure from any agent and reaches a posterior of $\eta(1 - \gamma, p, \frac{1}{2})$. Therefore, agent 1 wants to disclose $\bar{s}$, which forms a contradiction.

By strategic substitution, if agent 2 conceals more than two signals and agent 1 is believed to conceal four signals, agent 1 wants to disclose $\bar{s}$ as well. That is, there is no equilibrium where one agent conceals all four signals and the other agent conceals two or more than two signals.

Step 2. There is an equilibrium where both agents conceal three signals. Given that agent 2 conceals three signals and agent 1 is believed to conceal three signals, agent 1's payoff of concealing s^h is:

$$\gamma(1-\delta)p + (1-\gamma(1-\delta)p)\frac{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta)}{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta) + (1-\eta(\gamma,p,\frac{1}{2}))} = 0.490532 > \gamma - c$$

where $\eta(\gamma,p,\frac{1}{2}) = \frac{1-p+p\delta}{1-p+p\delta+1}$. If agent 2 discloses a signal (with probability $\gamma(1-\delta)p$), then it must be $\bar{s}$, which gives agent 1 a payoff of 1. Since the payoff of concealing s^h is greater than the payoff of disclosing it, agent 1 conceals s^h.

Step 3. There is no equilibrium where one agent conceals three signals and the other conceals less than three signals. First we claim that there is no equilibrium where agent 1 conceals two signals and agent 2 conceals three signals. Given that agent 2 conceals three signals and agent 1 is believed to conceal two signals, agent 1's payoff of concealing s^h is:

$$\gamma(1-\delta)p + (1-\gamma(1-\delta)p)\frac{\eta(1-\gamma,p,\frac{1}{2})(1-p+p\delta)}{\eta(1-\gamma,p,\frac{1}{2})(1-p+p\delta) + (1-\eta(1-\gamma,p,\frac{1}{2}))} = 0.485819 > \gamma - c.$$

Therefore, agent 1 conceals s^h, which rules out the existence of such an equilibrium. This further implies that there is no equilibrium where both agents conceal two signals.

Now suppose that agent 1 conceals two signals and randomizes on concealing s^h with probability $\lambda \in (0,1)$. Note that $\eta(\hat{s}_1,p,\frac{1}{2}) = \frac{1-p+p(1-\gamma)\delta+p\gamma\delta\lambda}{2-p+p\delta\lambda}$ is increasing in λ for the parameter case we consider. The payoff of concealing s^h is better than when he is believed to conceal only two signals ($\lambda = 0$). This implies that agent 1 also strictly prefers to conceal s^h.

Step 4. There are no equilibria where one agent conceals two signals and mixes on concealing the third while the other agent conceals three signals and mixes on concealing the fourth. WLOG, suppose agent 2 conceals two signals and mixes on the third in equilibrium and agent 1 conceals three signals and mixes on the fourth in equilibrium. Recall that, from Step 2, given agent 2 conceals two signals and agent 1 is believed to conceal four signals, agent 1 wants to disclose the fourth signal. Then, given that agent 2 conceals more than two signals and agent 1 is still believed to conceal four signals, strategic substitution implies that agent 1 wants to disclose the fourth signal as well. Now instead suppose that agent 1 is believed to conceal three signals and randomize on the fourth signal, i.e., less than four signals. Since $\eta(\hat{s}_1,p,\frac{1}{2})$ is strictly increasing over $\hat{s}_1 \geq \gamma$ and reaches its maximum at $\eta(1,p,\frac{1}{2}) = \frac{1}{2}$, agent 1's payoff of concealing the fourth signal is even worse than if agent 1 is believed to conceal four signals, so agent 1 wants to disclose the fourth signal. Therefore, the above proposed mixed strategy equilibrium does not exist.

Step 5. From Step 3 and Step 4, one learns that the best equilibrium for DM must be that both agents conceal three signals. In this equilibrium, the expected payoff of the DM is:

$$-(\frac{1}{2}(1-p(1-\delta))^2 + \frac{1}{2})\frac{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta)}{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta)+(1-\eta(\gamma,p,\frac{1}{2}))}$$
$$\left(1 - \frac{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta)}{\eta(\gamma,p,\frac{1}{2})(1-p+p\delta)+(1-\eta(\gamma,p,\frac{1}{2}))}\right) = -0.218215$$

In the best equilibrium under a single agent, i.e., the agent conceals two signals up to s^l, the expected loss of the DM is:

$$-(1-\frac{1}{2}p)\eta(1-\gamma,p,\frac{1}{2})(1-\eta(1-\gamma,p,\frac{1}{2})) + \frac{1}{2}p\delta\gamma(1-\gamma) = -0.21592$$

Therefore, comparing the best equilibrium, the DM is worse off with two upward biased agents than with only a single agent. □

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