Corporate Finance and the Monetary Transmission Mechanism*

Patrick Bolton† Xavier Freixas‡

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†Columbia University, 3022 Broadway, Uris Hall Room 804, New York, NY 10027; phone: (212) 854-9245; email: pb2208@columbia.edu.

‡Universitat Pompeu Fabra, Departament d’Economiques i Empresarials, C. Ramon Trias Fargas 25-27, Barcelona, 08005, Spain; phone: +34 93 542 2726; email: xavier.freixas@upf.edu.
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Abstract

We analyze the transmission effects of monetary policy in a general equilibrium model of the financial sector, with bank lending and securities markets. Bank lending is constrained by capital adequacy requirements and asymmetric information adds a cost to outside bank equity capital. In our model, monetary policy does not affect bank lending through changes in bank liquidity; but rather, it operates through changes in the spread of bank loans over corporate bonds, which induce changes in the aggregate composition of financing by firms, and in banks’ equity-capital base. The model produces multiple equilibria, one of which displays all the features of a “credit crunch.”

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1 Introduction

This paper is concerned with the monetary transmission mechanism through the financial sector, in particular the banking sector and securities markets. Specifically, it analyzes the effects of open market operations on bank lending and securities issues in a real economy. By building on recent advances in the microeconomics of banking it provides some underpinnings for the “credit view” of monetary policy, which, in its simplest form, relies on an exogenously assumed limited substitutability between bank loans and bonds.

The macroeconomics literature distinguishes between the “money view” and “credit view” of monetary policy transmission (Bernanke and Blinder, 1988 and 1992). The money view takes bonds and loans to be perfect substitutes and only allows for effects of monetary policy on aggregate investment, consumption, and savings through changes in interest rates. The credit view allows for an additional effect on investment and economic activity operating through bank credit supply controlled through changes in bank reserve requirements. Two critical assumptions underlying the credit view are that firms cannot easily substitute bank loans for bonds, and banks cannot substitute reservable liabilities (deposits) and non-reservable liabilities (bank bond issues).

Our contribution in this paper is to point to another channel operating through bank equity-capital. This channel works not only through the equilibrium composition of funding between direct and intermediated finance but
also through banks’ incentives to raise equity-capital. In our model banks are capital constrained in equilibrium since equity capital is more costly than other sources of funding like deposits or bonds. Banks are economizing on their cost of funding by holding no more than the required amount of equity capital. In addition, banks limit the size of their equity issues in an effort to economize their cost of capital.

The reason why equity capital has a higher cost than other sources of funding in our model is due to asymmetric information and information dilution costs as in Myers and Majluf (1984). That is, when a bank decides to raise additional equity through a seasoned offer, the market tends to under-value the issue for the better banks. But since it is the better banks that drive the decision whether to raise equity, the overall effect on all banks’ equity issues (whether good or bad) is to reduce the amount of equity raised relative to the full information optimum. Thus, because of information asymmetries about the true value of bank assets, there is an endogenous cost of equity, and by extension an endogenous cost of bank lending. Hence, banks’ equity base (which includes retained earnings) is a key variable in determining the total amount of bank credit.

An important consequence of this endogenous cost of equity is that multiple equilibria may exist. In one equilibrium the endogenous cost of capital (generated by self fulfilling market beliefs) is high while in the other it is low. The former has all the main features of a “credit crunch”\(^1\), namely that: i) bank lending is limited by a lower endogenous stock of bank capital; ii) there
is a correspondingly lower volume of bank credit, and; iii) equilibrium bank
spreads are high.\textsuperscript{2} In contrast, the other equilibrium has a high stock of bank
capital, a high volume of credit and lower equilibrium bank spreads.

Another way of thinking about this multiplicity of equilibria is in terms
of hysteresis in market beliefs about underlying bank values. Starting from a
low level of equilibrium bank capital, a single bank’s decision to issue equity
is likely to be interpreted by the market as a bad signal about the issuing
bank’s value (resulting in a reduction in the market price of bank equity),
thus inhibiting new equity issues. Vice-versa, in a situation where most banks
are expanding their capital base, a failure to expand will be interpreted as a
negative signal. This is the source of multiplicity of equilibria in our model.

This multiplicity of equilibria can give rise to potentially large monetary
policy transmission effects if a change in monetary stance induces a switch
from one equilibrium to another. One possible scenario, for example, is for a
tightening in monetary policy to push the economy from an equilibrium with
a high equity-capital base and high levels of bank lending into a credit crunch
equilibrium, with a low equity-capital base and low levels of bank lending.
How can this happen? One effect of monetary tightening in our model is to
reduce equilibrium bank spreads. Once those spreads hit a critically low level
it is no longer worth for banks to maintain a high equity-capital base. In
other words, the high equity-capital and high lending equilibrium is no longer
sustainable, and only the credit crunch equilibrium can arise for sufficiently
low bank spreads. If there is hysteresis, as described above, then once the
economy has settled in a credit crunch equilibrium, a major change in interest rates may be required to pull it out of this low lending equilibrium. That is, the economy may be stuck in the inefficient equilibrium as long as market beliefs are unchanged.

An important effect of monetary policy that our analysis highlights is related to the financial composition of the corporate sector between securities issues and bank credit. Recent empirical work suggests that one effect of monetary policy is to change firms’ financing decisions, with corporations substituting bank lending for commercial paper issues. A common explanation for these changes is that when bank cash reserves are tight, firms turn to the securities market to raise funds (see Kashyap and Stein (1994) and Gertler and Gilchrist (1994)). Our article, however, identifies a different and more complex transmission mechanism, which operates through bank equity-capital constraints as opposed to bank reserves.

The corporate finance side of our model builds on the more detailed analysis of our related paper (Bolton and Freixas, 2000). What distinguishes bank debt from corporate bond financing in our model is the flexibility of the two modes of financing: bank debt is easier to restructure but because bank capital (and therefore bank loans) is in short supply, there is an endogenous cost of flexibility. In other words, what makes bank loans expensive is the existence of a capital requirement regulation together with a dilution cost for outside equity. This, along with direct costs of running banks, is the source of the positive equilibrium spread between bank loans and bonds in
Firms with higher default risk are willing to pay this intermediation cost because they have a greater benefit of flexibility. This is where monetary policy affects the composition of financing: by raising real interest rates and thus lowering the equilibrium bank spread it induces some marginal firms to switch from bond financing to bank borrowing. This increase in the demand for bank loans is, however, offset by a decrease in bank lending at the other end of the risk spectrum to the riskiest firms, so that aggregate bank lending remains unaffected as long as bank equity-capital remains unchanged.

In our model, the impact of monetary policy on aggregate investment is thus more complex than in other models of the bank-lending channel. A tightening of monetary policy not only results in the usual increase in interest rates, but also gives rise to a decrease in spreads on bank loans. These two effects in turn induce a reduction in corporate securities issues and an improvement in the risk composition of bank loans. The latter effect is due to the fact that the riskier firms are ‘priced out’ of the bank credit market because, with lower spreads, bank loans are cheaper relative to bonds than before. As a result, some safer firms switch away from bonds to bank loans, thus ‘crowding out’ the riskier firms. The second major effect of a monetary tightening, as we show in proposition 4, is on bank equity-capital issues (and thus on overall bank lending) which are reduced due to the lower profitability of banks.

Most of the predictions of our model are consistent with empirical find-
ings in the banking literature. In particular, using a uniquely detailed data
set from the Bank of Italy, Gambacorta and Mistrulli (2004) find that cross-
sectional differences in bank lending responses to monetary policy relate to
differences in equity-capital constraints, thus providing support for a bank
capital channel. Also, Berger and Udell (1992) have shown that the spread
of commercial bank loans over Treasury rates (either nominal or real) is a de-
creasing function of the Treasury rates, as our analysis predicts. Their study
suggests a number of possible explanations for this finding. Interestingly,
although this is not our objective here, our paper suggests a new reason for
the observed commercial loan rates stickiness.

Several related recent papers also deal with monetary policy transmis-
sion through a bank lending channel. The four most closely related ones are
Gorton and Winton (1999), Van den Heuvel (1999), Schneider (1998) and
Estrella (2001). The first two papers focus on banks’ capital adequacy con-
straints and the macroeconomic effects of changes in bank lending induced by
changes in banks’ equity base. Bank capital is costly in Gorton and Winton
because bank equity is risky and requires both a risk and liquidity premium.
Capital adequacy constraints impose a cost on banks whenever investors’ op-
timal portfolio is less heavily weighted towards bank equity than is required
by regulations. This is most likely to occur in recessions. Accordingly the
amplification effects of monetary policy are greatest at the onset of a reces-
sion, when higher interest rates affect aggregate investment both directly and
indirectly through a reduction in bank lending capacity.
In Van den Heuvel it is assumed that there is an imperfect market for banks’ equity which implies that bank cannot readily raise new equity. Still, they can increase their capital stock through retained earnings. The amplification effects of monetary policy then work through their effects on retained earnings. While Van den Heuvel’s microeconomic model of banking is more rudimentary than that of Gorton and Winton his dynamic macroeconomic analysis goes considerably further, exploring lagged effects of changes in interest rates. In the same vein, Schneider provides an extensive dynamic macroeconomic analysis, which relies on a combination of liquidity and bank capital effects. Finally, Estrella provides a similar dynamic analysis focussing on the cyclical effect of Value at Risk regulation.

Neither of these models, however, allows for other sources of corporate financing besides bank lending and therefore cannot explore composition effects of monetary policy. Nor do these models allow for multiple equilibria and the possibility of what we describe as a “credit crunch” equilibrium, where bank lending is constrained by investors’ excessive pessimism about banks’ underlying asset values.

Romer and Romer (1990) observe that if banks are able to obtain funds by tapping financial markets, then monetary policy would affect banks only through changes in interest rates. There would be no specific bank lending channel. In response to Romer and Romer (1990), Lucas and McDonald (1992) and Stein (1998) have argued that non-deposit liabilities are imperfect substitutes for deposit liabilities (which are subject to reserve requirements)
when banks have private information about their net worth. They show that when certificates of deposit (CDs) are risky then banks are unable to perfectly substitute CDs for deposits, so that bank lending may be partially controlled by monetary authorities through changes in reserve requirements. Our model emphasizes instead the imperfect substitutability of equity capital with other sources of funds, and highlights that there is a bank lending channel operating through the bank equity-capital market even when banks have perfect access to the CD or bond market.

Since our model allows for the coexistence of bank lending and securities markets it is also related to a third set of papers by Holmstrom and Tirole (1997), Repullo and Suarez (2000) and Bolton and Freixas (2000), which all characterize equilibria where bank lending and direct financing through securities issues are both present. The latter three papers take intermediation costs to be exogenous and do not analyze the effects of monetary policy on bank equity-capital. A major difference with these papers is thus that our model allows for an endogenous level of bank capital. This is obtained at the price of some simplifying assumptions. Still, our model can be straightforwardly extended to allow for different initial levels of bank capital across banks.

The paper is organized as follows: Section 2 is devoted to the description of the model while Section 3 deals with banks lending and their asset/liability structure. Section 4 characterizes the general equilibrium when banks’ equity is fixed and Section 5 endogenizes the supply of bank equity and shows how
a “credit crunch” equilibrium may obtain. Section 6 considers comparative statics and the effects of monetary policy. Finally, Section 7 offers some concluding comments. The proofs of most results are given in an appendix.

2 The model

We consider a real economy with a single consumption or production good, which can be thought of as wheat. Monetary policy in this economy operates through open market operations (Government bond issues $G$) that affect interest rates on Government Bills. For simplicity, the nominal side of this economy is not explicitly modeled.

2.1 Firms’ investment projects and financial options

Each firm has one project requiring an investment outlay $I > 1$ at date $t = 0$. The project yields a return of $V > I$ when it succeeds. When it fails the project can generate a value $v$, as long as the firm is restructured. If the firm is unable to restructure its debts following failure then the value of the project is zero.

Firms’ owner-managers invest $W < I$ in the firm and must raise $I - W = 1$ from outside. Firms differ in the observable probabilities $p$ of success, where we assume that $p$ is uniformly distributed on the interval $[0,1]$.

Firms can choose to finance their project by either issuing bonds or by means of a bank loan. To keep the corporate financing side of the model to
its bare essentials, we do not allow firms to issue equity or to combine bonds and bank debt. The main distinguishing features of these two instruments are the following:

1. **bond financing**: a bond issue specifies a time $t = 1$ repayment to bond holders of $R(p)$. If the firm is unable to meet this repayments the firm is declared bankrupt and is liquidated. Restructuring of debt is not possible because of the wide dispersion of ownership of corporate bonds (see e.g. Bolton and Scharfstein (1996)).

2. **bank debt**: a bank loan specifies a repayment $\widehat{R}(p)$. If the firm defaults following failure of the project, the bank is able to restructure the firm’s debts and obtain a restructuring value of at most $v$.

As we will show, in equilibrium firms will be segmented by risk classes in their choice of funding, with all firms with $p \in (p^*, 1]$ choosing bond financing and all firms with $p \in [0, p^*]$ preferring a bank loan. Bond financing is preferred by low risk firms (with a high $p$) because these firms are less likely to fail at date $t = 1$ and therefore have less of a need for the costly debt restructuring services provided by banks. These services are costly because banks themselves need to raise funds to be able to lend to firms.

Having described the demand side for capital by firms we now turn to a description of the supply side.
2.2 Households

There is a continuum of households in our economy represented by the unit interval $[0, 1]$, with utility function

$$U(C_1, C_2) = \log(1 + C_1) + \log(1 + C_2)$$

Each household is endowed with one unit of good. For a given fixed gross real interest rate, $R_G$, aggregate investment and savings in this economy are determined by households’ optimal savings decisions. That is, households determine their optimal savings $s = (1 - C_1)$ by maximizing their utility function $U(C_1, C_2)$ subject to the budget constraint

$$C_1 + \frac{C_2}{R_G} = 1.$$

It is straightforward to check that households’ optimal savings function is then given by

$$s^* = 1 - C_1 = 1 - \frac{1}{2R_G}.$$

Households can invest their savings either in bank deposit accounts, in bonds issued by firms, in government bonds, or in bank equity. We denote the supply of deposits by $D(R_D)$ (with $0 \leq D(R_D) \leq 1 - \frac{1}{2R_G}$), where $R_D$ is the remuneration of deposits. We allow for perfect substitutability between deposit accounts and financial assets. For positive amounts of both deposits
and bonds, this will lead to the no arbitrage condition:

\[ R_G = R_D. \]

Given this no arbitrage condition, henceforth we refer to \( R_G \) as both the interest rate set on government bills and the remuneration on deposits. We will assume that bond returns and bank returns are independently distributed and that customers hold perfectly diversified portfolios. This simplifies the analysis by allowing us to model investments in an aggregate bond and bank equity portfolio as providing an essentially safe return. No arbitrage then also requires that the expected return on bonds \( pR(p) \) is such that:

\[ pR(p) = R_G. \] (1)

### 2.3 Banks

Banks, as firms, are run by self-interested managers, who have invested their personal wealth \( w \) in the bank. They can operate on a small scale by leveraging only their own capital \( w \) with (insured) deposits \( D \), so as to fund a total amount of loans \( w + D \). Their lending capacity will then be constrained by capital adequacy requirements:\9

\[ \frac{w}{w + D} \geq \kappa > 0. \]
Alternatively, banks can scale up their operations by raising outside equity-capital $E$ to be added to their own investment $w$. In that case their lending capacity expands from $\frac{w}{\kappa}$ to $\frac{w+E}{\kappa}$. However, when they raise outside equity they may face informational dilution costs. Outside equity investors, having less information about the profitability of bank loans will tend to misprice banks’ equity issues. In particular, they will underprice equity issues of the most profitable banks. We assume that banks choose an amount of equity to issue within the interval $[0, E]$, where $E < \infty$.

Because banks are perfectly diversified they have a zero probability of default, a simplifying assumption that allow us to sidestep the complexities associated with banks’ credit risk.

Banks face unit operating costs $c > 0$. These costs are best interpreted as operating costs banks must incur to attract depositors and potential borrowers, and may also be thought of as screening costs the bank incurs on each loan to determine the probability of success $p$. Thus, these costs are incurred, whether the bank ends up extending loans to firms or not.

To model bank dilution costs of equity-capital we take as a basic premise that bank managers differ in their ability to profitably run their bank. Specifically, we assume that bank managers may be either good or bad. Good bank managers (or type $H$-banks in our notation) are able to squeeze out a return $v$ from restructuring a defaulting firm, while bad bank managers (or $L$-banks) can only obtain a return $\beta v$, $(1 > \beta > 0)$. There are obviously other perhaps more plausible ways of modeling the difference between good and bad banks,
but the appeal of our formulation is its simplicity. Banks’ outside investors
do not know the bank’s type; all they know is that any bank they face is an
$L$-bank with probability $\mu$ and an $H$-bank with probability $1 - \mu$. This infor-
mational asymmetry about bank type gives rise to mispricing of each bank
type’s equity. It is the main source of costs of bank capital in our model.

A bank manager seeks to maximize his wealth and cares both about bank
profits and the bank’s share price. The reason why a bank manager cares
about share price is that he may need to sell his stake in the bank before the
returns of the bank’s loans are fully realized and known.

We model these objectives by assuming that bank managers may need to
liquidate their stake in the bank at date $t = 2^-$ with probability $\lambda \in (0, 1)$. Denoting by $q$ the share price of the bank and by $\Pi_2$ the bank’s accumulated profit up to period $t = 2$, the bank manager’s objective is then to maximize

$$\max [q, \lambda q + (1 - \lambda)\Pi_2].$$

If the manager is running an $L$–bank and he knows that $\Pi_2 < q$ (based on
his private information) then he will always sell his stake at date $t = 2^-$
and he will only care about the bank’s share price. If, on the other hand,
he is running an $H$–bank such that $\Pi_2 > q$ then he will seek to maximize
$\lambda q + (1 - \lambda)\Pi_2$.

Having determined the banks’ objectives, their investment opportunities
and their sources of funds, we now can turn to an analysis of their optimal
lending policy and asset-liability structure given fixed market terms.

Before doing so we briefly summarize the sequence of moves and events, and also recall the underlying information structure.

2.4 Timing

The following time line illustrates the order of decisions (see Figure 1).

1. At date $t = 1$

   • The Government sets $G$, announces an interest rate $R_G$ and firms issuing bonds quote their terms $R(p)$.

   • Banks quote their lending terms $\hat{R}(p)$ to firms and choose the amount of new equity they want to issue, $E$.

   • Firms who prefer bank lending apply for a loan, those preferring bond financing tap financial markets.

   • Banks make their portfolio decision. In particular, they decide what proportion of their funds to invest in new loans and what proportion in government or corporate bonds. These decisions are unobservable to investors.

   • Households determine the fraction of their endowment they want to save, and the proportion of their savings they want to hold as deposits and in direct investments.
2. At date $t = 2^-$ firms’ returns are realized. Those firms whose project has failed may be restructured if they have been financed with a bank loan.

Bank managers have the option to sell their equity stake in the secondary market.

3. At the end of date $t = 2$ all remaining debts as well as dividends are paid and households consume their net income.

### 3 Bank Lending and Optimal Asset-Liability Structure

As we shall establish, in our model only pooling equilibria (where $L$-banks mimic $H$-banks) exist.\textsuperscript{12} There may, of course, be many such equilibria. But, as we explain in Section 5, it is reasonable to focus on the pooling equilibria that are best for $H$-banks. Accordingly, we shall consider optimal lending and asset-liability management from the perspective of $H$-banks, who know that their observable actions are mimicked by $L$-banks (in a pooling equilibrium).

An $H$-bank contemplating an equity issue faces the following trade-off. If it issues equity it can increase lending and thus raise profits, but since its equity is undervalued in the financial market the bank’s manager does not appropriate the entire increase in profits. Depending on the profitability of loans and the extent of the undervaluation of equity the $H$-bank may or may
not decide to relax it’s lending constraint by issuing more equity. Thus, to
determine an $H$-bank’s choice we need to specify the profitability of loans
and the extent of dilution.

### 3.1 Optimal Lending Policy

In a pooling equilibrium $H$-banks quote lending terms $\hat{R}(p)$ to equalize the
expected profit on every loan they make. We denote by $\rho_H$ the expected net
excess return per loan for $H$-banks over government bonds. The reason why
the spread on each loan for an $H$-bank must be the same at an optimum is
that otherwise an $H$-bank could increase its profit by lending only to the firms
with the highest spread. The spread $\rho_H$ corresponds to the rents banks earn
because of the specific role their funding plays in restructuring and because
of the limited amount of capital they may have:

$$\rho_H = p\hat{R}(p) + (1 - p)v - R_G \quad (2)$$

Note that given these lending terms, $H$-banks get a higher return per loan
than $L$-banks. Indeed, the expected return on a loan with success probability
$p$ for an $L$-bank is only

$$p\hat{R}(p) + (1 - p)\beta v - R_G. \quad (3)$$

Given the expected net excess return per loan over government bonds
$\rho_H$ and given rates on government bonds and bank deposits of $R_G$, an $H$-
bank chooses its optimal mix of loans \( L \) and government bond holdings \( G_b \) to maximize expected profits subject to capital adequacy constraints:

\[
\begin{align*}
\max_{(L,G_b)} & \left\{ L(R_G + \rho_H - c) + (G_b - D(R_G))R_G \right\} \\
\text{subject to:} & \\
L + G_b &= D(R_G) + w + E \quad (AL) \\
L &\leq \frac{1}{\kappa}(w + E) \quad (\kappa)
\end{align*}
\]

where, \( (AL) \) is the asset-liability accounting identity and \( D(R_G) \) satisfies \( 0 \leq D(R_G) \leq 1 - \frac{1}{2R_G} \).

Note first that since the cost of raising funds through bank bond issues or bank deposits is the same, we are only able to determine the net amount of bonds minus deposits (positive or negative) the bank holds: \( G_b - D(R_G) \).

Second, it is easy to see from this program that the capital adequacy constraint is always binding if \( \rho_H > c \). Indeed, if the excess return on bank lending is strictly positive, an \( H \)-bank can always make a profit by raising an extra dollar and investing it in a bank loan. If, on the other hand, \( \rho_H < c \), then it is best for the bank not to lend at all to firms and to invest only in the market.
We summarize these observations in the lemma below.

**Lemma 1**: The optimal amount of lending for an $H$-bank is $L = \frac{(w+E)}{\kappa}$ if $\rho_H > c$ and $L = 0$ if $\rho_H < c$.

This lemma highlights that banks’ optimal lending policy and asset-liability structure is such that their equity capital base is always a binding constraint on their lending capacity. Thus, in response to an increase in spreads, banks can only increase lending if they also increase their equity-capital base. Thus, it becomes essential to consider how banks’ capital base is determined.

An important implication of this lemma is that monetary policy cannot affect bank lending by changing bank reserves (while keeping interest rates fixed). In other words, the classical bank lending channel of monetary policy is absent. As has already been noted by Romer and Romer (1990), when banks can perfectly substitute non reservable liabilities for reservable ones, as in our model, monetary authorities can no longer control bank lending by controlling bank reserves. Bond issues may, of course, be imperfect substitutes for insured deposits if there is a risk of default, as Stein (1998) has noted. However, this imperfect substitutability of risky bonds for safe deposits is only a necessary condition for bank liquidity to affect bank lending. It is not sufficient if capital adequacy constraints remain binding.

Indeed, if we introduce reserve requirements into our model, by requiring banks to hold a fraction $\varphi \in (0, 1)$ of their deposits as cash-on-hand
(remunerated or not remunerated), and if we introduce an imperfect substitutability between bonds and insured deposits by, say, adding a small spread \( \pi > 0 \) on bank bond issues (so that a dollar raised in bonds costs the bank \( R_G + \pi \)), then the effect would mainly be to raise the bank’s overall cost of funds. But as long as bank spreads \( \rho_H \) remain sufficiently high, banks would continue to raise funds up to the point where the capital constraint binds.

Thus, when the central bank changes reserve requirements on deposits, by changing \( \varphi \), the only immediate effect is on bank profitability. It cannot affect bank lending if the capital constraint remains binding. Of course, profitability eventually or indirectly affects the availability of capital, so that there may be an indirect or lagged effect on lending as in Van den Heuvel (1999).

These points are made more formally in Section 6 dealing with comparative statics.

Our result that bank equity capital is always a binding constraint on bank lending appears to be counterfactual, as banks generally have a higher capital base than is required by BIS regulations. However, we show in the appendix that the fact equity capital is higher than is strictly required at any point in time, does not necessarily mean that banks’ capital constraint is not binding. The point is that if banks anticipate that their role as providers of flexible financing requires extending future lending to firms (as part of their loan commitments) then they will hold capital reserves in anticipation of those future loan increases. This is why they may appear to be unconstrained, while in fact their equity base may actually constrain current lending. Intro-
ducing this idea formally into our model would have significantly increased its complexity. This is why we have chosen not to introduce it. Instead, we briefly describe the main changes to be made to the model to obtain a time 0 non-binding capital constraint in the appendix.

4 General Equilibrium in the credit market

In this section we take banks’ equity-capital as given and determine equilibrium rates $R_G$ and $\hat{R}(p)$ (or equivalently $R(p) = \frac{R_G}{p}$ and $\rho_H$) such that:

1. the aggregate demand for bank credit is equal to aggregate supply,

2. the aggregate demand for bank equity, corporate and government bonds is equal to the supply of funds to the securities markets,

3. bank demand for deposits equals deposit supply.

Since this last condition is met by setting $R_D = R_G$, for a given level of equity issues $E$, our equilibrium analysis boils down to solving a system of two equations in two unknowns, $R_G \geq 1$ and $\rho_H \geq 0$.13

The only difficulty in this analysis lies in constructing the aggregate demand and supply functions. Once these functions are determined, we can define two equilibrium schedules as functions of $R_G$ and $\rho_H$, one for the bank credit market and one for the securities markets. We then end up with a simple system of two equations as depicted in Figure 2, which can be solved straightforwardly.
Intuitively, to see why the credit market schedule is downward sloping note that any increase in $R_G$ which is not offset by a decrease in $\rho_H$ results in an overall increase in the cost of bank credit. This increase results in a drop in aggregate effective demand for bank credit, as the marginal riskiest firms get priced out of the market. But, for a constant $\rho_H$, aggregate supply of bank lending remains unchanged. Therefore, to maintain equilibrium in the credit market a fall in $\rho_H$ is required. Similarly, the securities market schedule is upward sloping because any increase in $R_G$ raises aggregate savings. When banks’ equity capital $E$ remains unchanged, banks’ demand for deposits remains unchanged following an increase in $R_G$, so that household demand for securities has to increase. To meet this increase in demand, firms must raise their supply of bonds, which in turn requires an increase in $\rho_H$ (see Figure 2).

4.1 Equilibrium in the bank credit market

We focus our analysis on equilibria in the credit market such that: (i) risky firms who cannot get a bank loan are also unable to get junk-bond financing and, (ii) all bank types lend up to capacity.$^{14}$

4.1.1 Firms funding choice

This equilibrium is such that all firms with a probability of default $(1 - p) > (1 - p^B)$ do not get any financing, all firms with a probability of default $(1 - p^B) \geq (1 - p) \geq (1 - p^*)$ get bank financing, and all firms with a very
low probability of default, \((1 - p^*) > (1 - p)\), get bond financing.

We now turn to a characterization of this equilibrium. Note first that, since firms do not appropriate any returns from restructuring, their demand for funds is simply driven by the cost of borrowing. A firm of risk characteristics \(p\) demands a bank loan if and only if

\[
\tilde{R}(p) - R(p) \leq 0
\]

Using equations (1) and (2), this is equivalent to:

\[
\frac{\rho_H - (1 - p)v}{p} \leq 0
\]

Therefore, any firm with a probability of success lower than the threshold

\[
p^* = \frac{v - \rho_H}{v}
\]

prefers a bank loan to a bond issue, and any firm with a probability of success larger than \(p^*\) prefers to issue bonds.

This is quite intuitive. Banks obtain a rent from restructuring firms. Their comparative advantage is therefore higher when they face a riskier firm, which is more likely to go through a restructuring.\(^{15}\)

While all firms with \(p < p^*\) apply for a bank loan not all of these will be granted one. Indeed, some of these firms may be too risky and have too low a rating \(p\) to be worth investing in.\(^{16}\) The threshold \(p^B\) below which firms
do not obtain credit is given by

\[ p^B V + (1 - p^B) v = R_G + \rho_H. \]  \hfill (4)

4.1.2 The demand for loans

Under our assumption that \( p \) is uniformly distributed on the unit interval, the mass of firms with \( p \leq p^B \), which cannot get any funding at the cost of funds \( (R_G + \rho_H) \) is given by: 17

\[ p^B (R_G + \rho_H) = \frac{R_G + \rho_H - v}{V - v} \]  \hfill (5)

And the aggregate demand for bank loans, comprising all firms with \( p \in [p^B, p^*] \), is given by: 18

\[ p^*(R_G, \rho_H) - p^B(R_G, \rho_H) = \left( \frac{v - \rho_H}{v} \right) - \left( \frac{R_G + \rho_H - v}{V - v} \right) \]

4.1.3 The supply of loans

Equilibrium in the bank credit market requires that the aggregate supply of bank credit \( L(R_G, \rho_H) \) equals this aggregate effective demand, \( p^*(R_G, \rho_H) - p^B(R_G, \rho_H) \).

When \( \rho_H > 0 \) all \( H \)-banks supply as much credit as they can given their equity-capital stock. As we pointed out above, this is not necessarily true for \( L \)-banks, however. These banks only prefer to lend to the corporate sector if
the return on their loans exceeds the return on bonds:

\[ p \hat{R}(p) + (1 - p)\beta v \geq R_G, \]

or

\[ \rho_H - (1 - p)(1 - \beta)v \geq 0. \]

Notice that, in contrast to \( H \)-banks, the expected net excess return per loan over government bonds for \( L \)-banks is higher for loans with a lower risk of default. Therefore \( L \)-banks concentrate on the safer segment of the bank-loan market and cover a risk-segment \([p^L, p^*]\), where \( p^L > p^H \) is defined by the equation

\[ p^* - p^L = \mu \left( \frac{w + E}{\kappa} \right), \quad (6) \]

when \( L \)-banks lend up to capacity. The \( RHS \) of this equation represents the maximum aggregate supply of loans by \( L \)-banks and the \( LHS \) is the aggregate demand for bank loans by the safest segment of firms seeking bank financing.

### 4.1.4 Equilibrium in the credit market

The aggregate supply of bank-credit in the equilibrium where all banks lend to capacity is given by \( L = \frac{w + E}{\kappa} \). In this equilibrium we have a schedule for the bank credit market given by:

**Lemma 2:** The equilibrium bank credit schedule relating \( \rho_H \) to \( R_G \) is
given by:

\[ \rho_H^C(R_G) = vA(E) - \frac{vR_G}{V}, \quad (7) \]

where:

\[ A(E) = \left( 1 - \left(1 - \frac{v}{V}\right) \frac{w + E}{\kappa} \right). \]

**Proof:** Equating supply and demand for bank loans we have

\[
\frac{w + E}{\kappa} = \frac{v - \rho_H}{v} - \frac{R_G + \rho_H - v}{V - v} \quad (8)
\]

or,

\[
\rho_H \frac{V}{V} = \left( 1 - \frac{w + E}{\kappa} \right) (V - v) + v - R_G
\]

Rearranging we obtain the desired expression.

Notice that the equilibrium schedule (7) is independent of \( G \). The reason is that banks do not compete directly with the government bond market in the equilibria we focus on. They only compete with the corporate bond market. Notice also that the schedule (7) defines a decreasing linear function in \( R_G \). That is, a higher \( R_G \) is associated with a lower equilibrium spread \( \rho_H \). The reason is that, with a fixed supply of bank loans (constrained by bank equity capital \( E \)), demand for bank loans can stay equal to supply only if an increase in \( R_G \) is partially offset by a decrease in \( \rho_H \). An equilibrium where \( L \)-banks lend to capacity and where aggregate loan-supply is \( L(\rho_H) = \frac{w + E}{\kappa} \) can be obtained when the equilibrium spread \( \rho_H^* \) is sufficiently high. We
provide a sufficient condition under which such an equilibrium exists below.

4.2 Securities market equilibrium

Securities markets clear when aggregate bond issues by the highest rated firms with total mass \((1 - p^*(R_G, \rho_H))\) together with government bond issues \(G\) and equity issues by banks \(E\) are equal to the aggregate supply of household savings (net of deposits) invested in financial markets \([1 - \frac{1}{2R_G} - D(R_G)]\) plus aggregate investments by the banking sector in government bonds \(G_b\). Equating aggregate demand and supply of securities we therefore obtain:

**Lemma 3:** The equilibrium securities market schedule relating \(\rho_H\) to \(R_G\) is given by:

\[
\rho_H^S(R_G) = vB(E) - \frac{v}{2R_G},
\]

where

\[
B(E) = 1 - \left( \frac{E}{\kappa} + w\left(\frac{1}{\kappa} - 1\right) + G \right).
\]

**Proof:** The securities market equilibrium condition is given by:

\[
1 - \frac{1}{2R_G} - D(R_G) = 1 - p^*(R_G, \rho_H) + E + G - G_b
\]

Replacing \(p^*(R_G, \rho_H)\) by its value and replacing \((D - G_b)\) by \((w + E)(\frac{1}{\kappa} - 1)\),
from the accounting identity \((AL)\), this condition becomes:

\[
\frac{v - \rho_H}{v} = \frac{1}{2R_G} + \frac{E}{\kappa} + w\left(\frac{1}{\kappa} - 1\right) + G
\]

Rearranging we obtain the desired expression.

Thus, the securities market equilibrium condition defines an increasing concave schedule \(\rho_H^S (R_G)\) parameterized by \(G\). The reason why the schedule is increasing in \(R_G\) is that any increase in \(R_G\) raises household savings. To be able to invest these increased savings in corporate bonds there has to be an equivalent increase in corporate bond issues. These issues, in turn, can only come from firms that otherwise would have taken out a bank loan. Thus, to get these firms to switch away from bank loans to bond issues there has to be an increase in the relative cost of bank loans; that is, an increase in \(\rho_H\). Similarly, the reason why the schedule is concave in \(R_G\) is that the household savings function is concave in \(R_G\).

Finally, notice that an increase in \(G\) induces a downward shift in the equilibrium schedule (9). The reason is that the corporate bond market also competes for household savings against the government bond market. Therefore, any increase in government bond issues must be met in part by an increase in savings (requiring in turn an increase in interest rates \(R_G\)), and by a contraction in corporate bond issues (requiring a reduction in \(\rho_H\)).

A general equilibrium in the securities and bank loan markets \((\rho_H^*, R_G^*)\) is obtained when the two functions \(\rho_H^C (R_G)\) and \(\rho_H^S (R_G)\) intersect (as shown
in Figure 2). In the next subsection we give two sufficient conditions which guarantee the existence of a unique general equilibrium with maximal bank lending.

### 4.3 Existence

We now establish that a unique general equilibrium \((\rho^*_H, R^*_G)\) with maximum bank lending exists if the following two conditions hold.

**Assumption A1**: 
\[
\frac{1}{2} - \frac{1}{V} + G > w - \frac{w}{V} \left(\frac{w}{\kappa}\right)
\]

**Assumption A2**: 
\[
w - \left[1 + \mu \left(\frac{1 - \beta}{\beta}\right)\right] \left(\frac{w + E}{\kappa}\right) > G - \frac{1}{2}
\]

The first condition guarantees that the two functions \(\rho^C_H (R_G)\) and \(\rho^S_H (R_G)\) intersect for some \(R_G \geq 1\). The second condition guarantees that the equilibrium spread is sufficiently high that both \(H\) and \(L\)–banks want to lend to capacity. As is easily seen, these two conditions are satisfied within a range of \(G\). If \(G\) is too low the two functions \(\rho^C_H (R_G)\) and \(\rho^S_H (R_G)\) may intersect only for \(R_G < 1\). But we must have \(R_G \geq 1\) for households to invest any savings in firms. Similarly, if \(G\) is too large the second condition is violated. In that case government borrowing is so large and equilibrium interest rates \(R_G\) so high that \(\rho_H\) is too low to make it profitable for \(L\)–banks to lend to the corporate sector.

The two conditions also hold for a range of bank equity-capital, \(w\) or
If bank-equity capital is too large bank loan supply to the corporate sector is so large that the loan market cannot clear with a spread $\rho_H$ that is high enough to induce $L-$banks to engage in maximal lending to the corporate sector. Note, finally, that assumption A2 is more likely to hold for a lower $\mu$ and a higher $\beta$. This is again easy to understand intuitively. A lower $\mu$ means that the mass of $L-$banks is smaller. Other things equal, therefore, any $L-$bank is able to lend to a better risk-pool ($p_L$ is higher), which raises the return on corporate lending. Similarly, a higher $\beta$ raises the return on lending for $L-$banks.

**Proposition 1:** Under assumptions A1 and A2 a unique maximum bank-lending equilibrium ($R_G^*, \rho_H^*$) exists.

**Proof:** Notice first that for $R_G$ sufficiently large we always have $\rho_H^C (R_G) < \rho_H^S (R_G)$ (this is obvious from Figure 2). Next we will prove that under assumption A1 $\rho_H^C (1) \geq \rho_H^S (1)$, and when this inequality holds the two equilibrium schedules can only intersect at some $R_G \geq 1$. To show $\rho_H^C (1) \geq \rho_H^S (1)$, subtracting equation (9) from (7) we obtain the:

$$A - B - \frac{1}{V} + \frac{1}{2} \geq 0$$

and, substituting for the values of $A$ and $B$,

$$\frac{1}{2} - \frac{1}{V} + G \geq w - \frac{v}{V} \left(\frac{w + E}{\kappa}\right).$$
an inequality implied by assumption A1.

Second, when assumption A2 holds we have $\rho_H^C(1) \geq (1 - p_L)(1 - \beta)v$ so that even at a spread $\rho_H^C(1)$, $L-$banks prefer to lend up to capacity to the corporate sector. A fortiori, then they are lending to capacity at the equilibrium spread $\rho_H^C(R_G^*) > \rho_H^C(1).^{19}$

Having established the existence of a unique general equilibrium for a fixed equity-capital base for banks $(w + E)$ that satisfy assumptions A1 and A2 we now turn to the endogenous determination of banks’ equity-capital. To guarantee the existence of an equilibrium with endogenous equity issues we shall take it that assumptions A1 and A2 hold for all $E \in [0, \bar{E}]$.

5 Endogenous bank equity

In this section we take into account banks’ incentives to issue equity and allow for the endogenous determination of bank-equity capital. Banks’ incentives to issue equity depend on the equilibrium beliefs of investors. We therefore face the standard equilibrium problem of the joint determination of equilibrium strategies and beliefs.

When banks must pay a premium for equity capital, they will expand their equity base only if the rate of return on bank-loans exceeds the cost of equity-capital. This is why equilibrium bank-spreads $[p\hat{R}(p) + (1 - p)v - R_G]$ will be strictly greater than banks’ average operating costs, $c$. Also, given that bank-spreads are strictly positive in equilibrium, banks have an incentive
to lend up to the point where the capital constraint binds.\textsuperscript{20}

As is well known, signalling games generally have multiple equilibria. We argue in this section that this observation may have important implications for equilibrium bank lending and for the monetary transmission mechanism. Indeed, we show that a high-lending equilibrium, with low rationally expected dilution costs and low equilibrium bank-spreads, may exist along with a low-lending credit-crunch equilibrium, with high spreads and high rationally expected dilution costs. In this context, even a small change in interest rates $R_G$ induced by monetary policy can have large effects on aggregate bank lending, if it induces a switch from the high-lending equilibrium to the low-lending equilibrium, or vice-versa.

Ultimately the relevant equilibrium is tied down by market beliefs and, as Spence (1974) has compellingly argued, a complete theory of how market beliefs are formed involves historical, psychological and cultural considerations, which go beyond the scope of our analysis. We can, however, narrow down somewhat the equilibrium set by appealing to intuitive refinement ideas along the lines of Cho and Kreps (1987). Nevertheless, as we show in this section a fundamental multiplicity will remain even after elimination of equilibria that are supported by market beliefs that do not satisfy Cho-Kreps’ intuitive criterion. We believe that this is a strength and not a weakness of the theory, as it provides the underpinnings for the notion of a “credit crunch” equilibrium.

We begin the section by considering a single bank’s incentive to issue new
equity given equilibrium rates of return on lending $\rho_H$ and $\rho_L$. We then derive the optimal amount of equity banks choose to issue, the aggregate supply of bank credit, and credit spreads in a general equilibrium of the capital and bank credit market.

In our equilibria, $H$-banks issue equity only if the bank spread $\rho_H$ is high enough and bank loans are sufficiently lucrative to compensate for the dilution cost on new equity issues. As for $L$-banks, they prefer to mimic $H$-banks to benefit from better prices on the sale of their shares.

5.1 Bank Owner-Manager Payoffs and Optimal equity issues

To be able to determine a bank’s incentive to issue equity we need first to derive the return on equity and bank manager’s payoff functions for each type of bank.

Characterizing the return on equity for each bank type $J = H, L$ turns out to be straightforward for a type-$H$ bank, but somewhat more involved for a type-$L$ bank. To simplify our notation we shall denote the return on equity-capital for a type-$J$ bank by $\Gamma_J$. Both types of bank must incur a cost per asset unit of $c > 0$, and $H$-banks earn a unit spread of $\rho_H$ over $R_G$ on each loan they make, while $L$-banks earn only $p\hat{R}(p) + (1 - p)\beta v - R_G$ on a loan to a firm with a probability of success $p$. Therefore, we obtain the following characterization for the return on equity for each bank type.
Lemma 4: The return on equity-capital for respectively $H$-banks and $L$-banks is given by

$$
\Gamma_H = R_G + \frac{\rho_H - c}{\kappa} \quad \text{and} \quad \Gamma_L = R_G + \frac{\rho_L - c}{\kappa},
$$

where

$$
\rho_L(\rho_H) = \rho_H \beta - Z, \quad \text{and} \quad Z = (1 - \beta)v \frac{\mu}{2} \left( \frac{w + E}{\kappa} \right).
$$

The returns $\Gamma_H$ and $\Gamma_L$ are the actual returns for each type of bank. Investors, however, do not observe banks’ types and can only draw inferences about a bank’s likely type given the bank’s observed actions, in particular the bank’s observed equity issue $E$. We shall denote by $\Gamma_E$ market beliefs about a bank’s type, conditional on the bank issuing equity $E$.

We now turn to the characterization of each bank type’s optimal equity issue decision given market beliefs $\Gamma_E$. A bank raising an amount of equity $E$ from new shareholders must relinquish equity ownership $\alpha$ equal to

$$
\alpha \Gamma_E(w + E) = E \cdot R_G. \quad (11)
$$

Recall that bank-managers face liquidity shocks which force them to unwind their equity holdings with probability $\lambda$ at date $t = 1$. Therefore, as highlighted in Section 2.3, an $H$-bank owner-manager’s expected payoff
following a new equity issue $E$ is given by:

$$(1 - \alpha)[\lambda \Gamma_E + (1 - \lambda)\Gamma_H](w + E).$$

Under the same market expectations, an $L$-bank manager’s expected payoff following a new equity issue $E$ is given by:

$$(1 - \alpha)\Gamma_E(w + E).$$

The expressions for the payoffs of the two types differ because a manager of an $L$-bank is always better off selling his equity stake at date $t = 1$ than holding on to it, since the market always (weakly) overvalues the shares of an $L$-bank.

Now, if we denote by $\Gamma_0$ the market’s expected return on equity-capital when a bank issues no equity ($E = 0$) and by $\Gamma_E$ the market expected return on equity-capital when a bank raises an amount of equity $E > 0$, each type of bank’s optimal equity issue decision is given by:

**Lemma 5:** An $H$-bank manager is better off issuing equity $E > 0$ than issuing no equity if and only if,

$$\frac{\Gamma_E(w + E) - E R_G}{\Gamma_E}[\lambda \Gamma_E + (1 - \lambda)\Gamma_H] \geq [\lambda \Gamma_0 + (1 - \lambda)\Gamma_H] w.$$  \hspace{1cm} (12)

An $L$-bank manager is better off issuing equity $E > 0$ than issuing no equity
if and only if,

$$\Gamma_E(w + E) - ER_G \geq \Gamma_0 w. \quad (13)$$

As will become clear in the next section, conditions (12) and (13) will be central for the determination of the full equilibrium with endogenous bank equity-capital.

For future reference it is helpful to put more structure on market conditional beliefs $\Gamma_E$. To that end, recall the timing of moves in our model: banks begin by quoting lending terms $\hat{R}(p)$ to firms and announcing new equity issues, $E$. Households then respond by deciding how they want to allocate their savings among the different financial instruments, all yielding an expected return of $R_G$ in equilibrium. When evaluating the return on bank-equity, households will be able to update their beliefs about a bank’s type given the observed policy of the bank, $\hat{R}(p)$ and $E$. Thus, if we denote by $\delta_H(\hat{R}(p), E)$ and $\delta_L(\hat{R}(p), E)$ investors’ updated beliefs about the bank’s type we have:

**Lemma 6:** The return on equity-capital $\Gamma_E$ expected by investors in equilibrium is given by

$$\Gamma_E = R_G + \frac{\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L - c}{\kappa} \quad (14)$$
5.2 Pooling Equilibria in the banking sector

We now turn to the derivation of the aggregate equilibrium supply of bank loans, by characterizing the Bayesian-Nash equilibrium of the equity-capital issue game banks play.

**Definition:** A Bayes-Nash Equilibrium in the banking sector given $R_G$ and $\hat{R}(p)$ is characterized by:

- Banks’ best equity issue strategies, $E^i \in [0, \bar{E}]$, $i = H, L$ given equity-market conditional beliefs, $\delta_H(\hat{R}(p), E)$ and $\delta_L(\hat{R}(p), E)$, and
- equity-market conditional beliefs about the bank’s type that are consistent with the banks’ best responses; that is, equity-market conditional beliefs that are consistent with Bayesian updating.

As one might expect, there may be many Bayes-Nash Equilibria in our game. We shall show, however, that separating equilibria such that $E^H \neq E^L$ do not exist. The intuition for this result is as follows. Two different types of separating equilibria are conceivable. One where the $H$-bank manager chooses not to issue equity ($E^H = 0$) while the $L$-bank manager issues new equity ($E^L > 0$). The other type of separating equilibrium has the $L$-bank and $H$-bank managers switch roles.

It is easy to see that the latter type of separating equilibrium cannot exist. The reason is simply that the $L$-bank manager would have a profitable deviation by mimicking the $H$-bank’s strategy. This would not only raise
the L-bank’s stock price but also allow the bank to sell overvalued equity. Symmetrically in the former type of separating equilibrium, for an H-bank to decide not to issue equity in a separating equilibrium in which it incurs no dilution costs à la Myers-Majluf, it must be the case that loans have a very low return (namely, \( \rho_H < R_G \)). But then it is even less profitable for L-banks to lend. Therefore, neither H nor L-banks have an incentive to issue equity.

**Proposition 3:** There exist no separating Bayes-Nash equilibria in the banking sector, such that \( E^H \neq E^L \), \( \delta_H(\hat{R}(p), E^H) = 1 \) and \( \delta_L(\hat{R}(p), E^L) = 1 \).

**Proof.** See the appendix.

Pooling equilibria can be supported by out-of-equilibrium beliefs such that \( \delta_L(\hat{R}(p), \hat{E}) = 1 \) for all \( \hat{E} \neq E \).\(^{23}\) A pooling equilibrium with bank equity-capital issues \( E \) is characterized by equilibrium spreads \( \rho_H(E) \), \( \rho_L(E) \) and interest rates \( R_G(E) \) given by the unique solution to the equilibrium equations (9) and (7) such that \( R_G(E) \geq 1 \).\(^{24}\)

As is to be expected, a continuum of pooling equilibria characterized by the size of the equilibrium equity issue \( E \leq \bar{E} \) may exist, since out-of-equilibrium beliefs can be set arbitrarily. We shall, however, restrict attention to the pooling equilibria that are best from the point of view of an H-bank manager, partly on the grounds that the Cho and Kreps intuitive criterion would select these equilibria over all other pooling equilibria in our game (see Cho and Kreps, 1987).\(^{25}\)
The optimal equity issue for a type $H$-bank, in a pooling equilibrium with lending terms $\hat{R}(p)$ and $R_G$, is generically either 0 or $E$, as is established in the following lemma.

**Lemma 7:** Given lending terms $\hat{R}(p)$ such that $p\hat{R}(p) + (1 - p)v - R_G = \rho_H$ the optimal equity issue for an $H$-bank under pooling is either:

- $E$, if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L > c$ or,
- 0 if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L < c$ or,
- undetermined if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L = c$.

**Proof.** We compute the sign of the derivative with respect to $E$ of the managers profit. The manager profit’s value is $V_H(E) = \frac{\Gamma_E w + E}{\Gamma_E} [\lambda \Gamma_E + (1 - \lambda) \Gamma_H]$, and its derivative’s sign is therefore given by the sign of $\Gamma_E - R_G$, which value we are able to compute as:

$$\Gamma_E - R_G = \frac{\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L - c}{\kappa}$$

which allow us to conclude. ■

Consider first the pooling equilibrium with $E^H = E^L = 0$. This equilibrium admits spreads $\rho_H(0)$, $\rho_L(0)$ and interest rates $R_G(0) \geq 1$, given by the unique solution to the equations

$$\rho_H = vA(0) - \frac{vR_G}{V}, \quad (15)$$

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and
\[ \rho_H = vB(0) - \frac{v}{2R_G}. \] (16)

Similarly, the pooling equilibrium with \( E^H = E^L = E \) admits spreads \( \rho_H(E), \rho_L(E) \) and interest rates \( R_G(E) \geq 1 \), given by the unique solution to the equations
\[ \rho_H = vA(E) - \frac{vR_G}{V}, \] (17)
and
\[ \rho_H = vB(E) - \frac{v}{2R_G}. \] (18)

When does either of these equilibria obtain? And, can both equilibria coexist? The next proposition provides sufficient conditions under which a pooling equilibrium always exists and under which both may coexist.

**Proposition 4:** Under assumptions \( A_1, A_2 \) and

\[ A_3 : \beta < \frac{(1 - \mu)\rho_H(E)}{\rho_H(0) - \mu \rho_H(E)} \]

a pooling equilibrium always exists. When, in addition assumption

\[ A_4 : \rho_H(0)\beta < \min\{c + (1 - \beta)v\frac{\mu w}{2\kappa}; (1 - \mu + \mu \beta)\rho_H(E)\} \] (19)

holds, both pooling equilibria may coexist.

**Proof.** See the Appendix. ☐
We have imposed three conditions to guarantee the existence of at least one and possibly multiple pooling equilibria with maximal bank lending. The reader may wonder at this point whether the set of parameters that satisfy assumptions $A_1$, $A_2$ and $A_3$ is not an empty set. Careful inspection of these three conditions reveals that the first two conditions impose a restriction on $G$ while the third one restricts the values of the parameter $\beta$. In the appendix we provide a numerical example in which both pooling equilibria with maximal bank lending do coexist. Note also that the range of parameter values for which pooling equilibria with maximal bank lending exist is larger than the set of parameters that satisfy assumptions $A_1$, $A_2$ and $A_3$, as these are sufficient conditions.

Nevertheless, the reader may be concerned about the robustness of our equilibria. The reason why we require $A_1$, $A_2$ and $A_3$ is that we want to focus on a specific type of equilibrium where $L$–banks lend to full capacity.

Clearly, assumption $A_2$ guaranteeing that $L$-banks want to engage in maximal lending to firms is unnecessarily restrictive. We only focus on this equilibrium because it is the simplest to characterize. But an equally plausible equilibrium could be one where $L$-banks do not lend up to capacity. An interesting and empirically relevant comparative statics feature of this equilibrium is that it produces an aggregate bank credit supply function (by $L$-banks) that is downward-sloping in $R_G$. Indeed, the higher $R_G$ the lower is $\rho_L$, and, therefore, the lower is the credit supply of $L$-banks. Assumption $A_1$, on the other hand, is quite natural as we do not expect to obtain
an equilibrium if the amount of government bond issues is not sustainable. Finally assumption A3 simply allows us to disregard semi-separating equilibria which are more cumbersome and do not bring any additional insight (see Figure 3).

The coexistence of two pooling equilibria is of interest for the study of the effects of monetary policy. Figure 3 illustrates this case. When the economy is initially in the high lending equilibrium with $E^H = E^L = \bar{E}$, a contractionary monetary policy resulting in an increase in $R_G$ and a reduction in bank spreads $\rho_H$ may induce a switch from the high-lending equilibrium to the low-lending one characterized by $E^H = E^L = 0$ and thus lead to a magnified contractionary effect. In other words, a contractionary monetary policy may drive the economy into a region of the parameter space, where only the pooling equilibrium such that $E^H = E^L = 0$ exists. This switch is inevitable if spreads $\rho_H(\bar{E})$ are reduced to the point where an $H$-bank finds it optimal to deviate from the high lending equilibrium policy ($E^H = E^L = \bar{E}$) by setting $E^H = 0$.

To state this point more formally, if initially the economy is characterized by a sufficiently low $R_G(\bar{E})$ and a sufficiently high $\rho_H(\bar{E})$ to satisfy:

$$\frac{\Gamma_{\bar{E}}(w + \bar{E}) - \bar{E} \cdot R_G(\bar{E})}{\Gamma_{\bar{E}}} \left[ \lambda \Gamma_{\bar{E}} + (1 - \lambda) \Gamma_H^T \right] \geq \left[ \lambda \Gamma_L^T + (1 - \lambda) \Gamma_H^T \right] w \quad (20)$$

where,

$$\Gamma_{\bar{E}} = \bar{\Gamma} = \mu \Gamma_L^T + (1 - \mu) \Gamma_H^T,$$
\begin{align*}
\Gamma^E_H &= R_G(E) + \frac{\rho_H(E)}{\kappa} - c \\
\Gamma^E_L &= R_G(E) + \frac{\rho_H(E)\beta - (1 - \beta)\psi\left(\frac{w + E}{\kappa}\right)}{\kappa} - c,
\end{align*}

and then, a severe monetary contraction may increase \( R_G(E) \) and lower \( \rho_H(E) \) to the point where:

\[
\frac{\Gamma^E(w + E) - E \cdot R_G(E)}{\Gamma^E} \left[ \lambda \Gamma^E + (1 - \lambda)\Gamma^E_H \right] < \left[ \lambda \Gamma^E_L + (1 - \lambda)\Gamma^E_H \right] w, \tag{21}
\]

in which case it is optimal for banks to deviate from the high lending equilibrium policy by setting \( E^H = E^L = 0 \).

We interpret the switch from the equilibrium with high bank-equity capital to the one with low equity-capital as a form of credit crunch induced by a bank equity crunch similar to the credit crunch in some Northeastern States in 1990 described by Bernanke and Lown (1991).

To summarize, a key prediction of our analysis in this section is that a monetary tightening not only brings about the expected increase in the cost of borrowing \( R_G \) and \( R_G + \rho_H \), but as it induces a decrease in bank spreads \( \rho_H \), it also reduces banks’ incentives to increase their equity-capital base. Capital-adequacy regulations thus may induce a decrease in the supply of loans through a contraction of the equity-capital base and thus magnify the contractionary effect of monetary policy.
6 Comparative Statics and the Effects of Monetary Policy

Having highlighted how changes in interest rates may produce a switch from a high bank lending equilibrium to a credit crunch equilibrium, we now turn to a discussion of ‘local’ effects of monetary policy by exploring the comparative statics of changes in $G$. We shall illustrate these comparative statics effects in a diagram and we confine the formal analysis to the appendix. What we have in mind here is a Central Bank holding a stock of T-bills and conducting monetary policy by buying and selling these bills against cash. An expansionary monetary policy is then implemented by buying T-bills (decreasing $G$) and a contractionary one by increasing $G$.

The effect of an increase in $G$ is displayed in Figure 4 below. When the supply of bonds increases from $G$ to $G'$, it does not affect the credit market equilibrium schedule, as equation (7) is independent of $G$. However, it shifts the security market equilibrium schedule (9) outward, as government borrowing crowds out corporate and bank bond issuers. One expected effect of increased government borrowing is an increase in interest rates, $R_G$. The other effect in our model is a decrease in bank spreads $\rho_H$. The reason why bank spreads must decrease is that otherwise total demand for bank loans would decrease to a point where banks would not be able to lend to capacity. But then it would be profitable for banks to lower $\rho_H$ slightly to be able to lend more (see Figure 4).
These effects on $\rho_H$ and $R_G$ in turn affect the equilibrium level of bank lending and bond issues by shifting overall financing to safer firms. That is, both $p(R_G + \rho_H)$ and $p^*_1(R_G + \rho_H)$ shift to the right by an amount proportional to the increase in $G$ (see the appendix), so that total bank lending remains unchanged, but the increased supply of T-bills partially crowds out corporate bond issues. It does not fully crowd out corporate bond issues since the increase in $R_G$ also induces an increase in savings.26

Thus, the effect of a monetary tightening on individual firms is to cut off the riskiest firms from bank lending and to induce substitution of bond financing for relatively cheaper bank lending at the firms with the lowest bond ratings. Overall, the total share of bank lending to corporate bond issues increases in response to a monetary tightening. We summarize these findings in the following proposition.

**Proposition 5: Effects of open market operations:** A contractionary monetary policy (increase in $G$) has the effect of: i) increasing T-bill rates $R_G$, ii) decreasing bank spreads $\rho_H$, iii) increasing the overall cost of bank loans $R_G + \rho_H$, iv) decreasing corporate bond issues by an amount that is smaller than the additional amount of government borrowing and, v) leaving the aggregate amount of bank lending unchanged. However, bank lending is now directed to safer firms, so that on balance it is the marginal firms with the highest risks that are forced out of investment by the tightening in monetary policy.
Proof. See the appendix.

Thus, for the pooling equilibria with maximal bank lending we have focused on, the only local effect of a monetary tightening and reduction in bank spreads is to change the composition of financing. Aggregate bank lending remains unchanged, but there is a shift away from bond financing towards borrowing through bank loans. However, when equilibrium bank spreads decrease to the point that assumption A2 no longer holds then a further increase in $R_G$ also induces $L$—banks to switch away from direct lending to the corporate sector to investing in securities. This switch will then result in a contraction of aggregate bank lending.

How do these results relate to empirical findings? The recent empirical literature on the monetary transmission mechanism has uncovered one broad finding on the composition effects of a contractionary policy. Kashyap, Stein and Wilcox (1993) have found that an important response to a monetary tightening is a surge in commercial paper issuance. They interpret this finding as a change in the composition of financing by firms in response to a monetary tightening: firms substituting bank debt for commercial paper. However, Gertler and Gilchrist (1994) and Oliner and Rudebusch (1995) find that the main factor behind this surge in commercial paper issuance is inventory build-up by large firms financed by commercial paper issuance. Small firms do not rely on commercial paper issues at all. Moreover, these firms bear the brunt of the monetary tightening. Thus, the story that seems to emerge from these studies is that:
“...the main effect of a monetary contraction is to shift financing of all types from small firms to large firms. This shift produces a decline in the aggregate bank-loan share because large firms rely less heavily on bank debt than do small firms.” [Oliner and Rudebusch, pp 301]

Our results are consistent with these findings to the extent that they explain the shift in overall financing from small (or riskier) firms to large (or safer) firms that results from an increase in interest rates $R_G$. They seem, however, in contradiction with the empirical evidence to the extent that in our model the aggregate bank loan share can increase as a result of the monetary tightening. But, note that our results concern the aggregate share of bank lending to long-term bond issues and not the share of bank lending to commercial paper or other short term debt, which is the focus of most of the empirical literature. The only study that investigates the ratio of bank loans to long term bonds is Gertler and Gilchrist (1983). They show that this ratio only declines slightly following a tightening in monetary policy. 27

Note also that our result that $\rho_H$ decreases in response to a monetary tightening is consistent with the stylized fact that the yield on bank loans is sticky relative to the Treasury rate. Our model thus provides an alternative explanation for the observed stickiness of bank loan rates.

Other comparative statics effects could be explored, such as the effects of a tightening in capital regulations (an increase in $\kappa$) a change in the quality
composition of banks (a change in $\mu$, or a change in $\beta$) a change in the profitability of investment projects (a change in $V$ or $v$). For lack of space we omit the discussion of these effects. But, it should be clear that the analysis of the comparative statics with respect to $G$ can be easily adapted to explore these effects.

7 Conclusion

This paper proposes a model of the interface between corporate financing decisions and monetary policy in a general equilibrium model (of the capital market), which traces the effects of monetary policy on firms’ investment decisions.

The model developed here, which abstracts from many other relevant considerations generates several qualitative predictions about the joint equilibrium in the credit and securities markets and the effects of open market operations on the real sector, which are broadly consistent with stylized facts on the effects of monetary policy on investment and firm financing uncovered by recent empirical studies.

The model considered in this paper is already somewhat complex and we have chosen to leave some interesting extensions for future research. An obvious immediate extension is to differentiate firms according to both the underlying risk of their cash flows and their size. If, in addition, one then introduces a fixed issuing cost for bonds (representing legal and administrative
costs) we would expect to obtain an equilibrium segmentation where only the largest and safest firms issue bonds. Such a model could also be used to investigate how the size distribution of firms as well as the relative costs of securitization affect the aggregate composition of financing in the economy.

Another obvious but more ambitious extension is to introduce a final goods market and sticky prices. Extending the model to a multiperiod setting in order to explore the dynamics of the monetary transmission mechanism is perhaps the most interesting and difficult challenge.

8 Mathematical Appendix

Proof of Proposition 3: The two functions $\psi_H$ and $\psi_L$ below represent the net payoff of issuing equity worth $E = \bar{E}$ instead of $E = 0$, for respectively an $H$ and an $L$-bank:

\[
\psi_H(\Gamma_E, \Gamma_0) = (\Gamma_E - R_G) \bar{E} \left[ \lambda + (1 - \lambda) \frac{\Gamma_H}{\Gamma_E} \right] - \lambda (\Gamma_0 - \Gamma_E) w
\]  

and

\[
\psi_L(\Gamma_E, \Gamma_0) = (\Gamma_E - R_G) \bar{E} - (\Gamma_0 - \Gamma_E) w
\]  

Combining expressions (22) and (23) yields the following convenient expres-
\[ \psi_H(\Gamma_E, \Gamma_0) = \lambda \psi_L(\Gamma_E, \Gamma_0) + (\Gamma_E - R_G) E (1 - \lambda) \frac{\Gamma_H}{\Gamma_E} \] (24)

The different types of equilibria are characterized by the values of the functions \( \psi_H \) and \( \psi_L \), as follows:

1. The pooling equilibrium with \( E = 0 \) is such that \( \psi_H(\Gamma_E, \Gamma_0) \leq 0 \) and \( \psi_L(\Gamma_E, \Gamma_0) \leq 0 \).

2. The pooling equilibrium with \( E = E \) is such that \( \psi_H(\Gamma_E, \Gamma_0) \geq 0 \) and \( \psi_L(\Gamma_E, \Gamma_0) \geq 0 \).

3. Separating equilibria are such that \( \psi_H(\Gamma_E, \Gamma_0) > 0 \) and \( \psi_L(\Gamma_E, \Gamma_0) \leq 0 \) or \( \psi_H(\Gamma_E, \Gamma_0) \leq 0 \) and \( \psi_L(\Gamma_E, \Gamma_0) > 0 \).

4. Semi-separating equilibria are such that either \( \psi_L(\Gamma_E, \Gamma_0) = 0 \) or \( \psi_H(\Gamma_E, \Gamma_0) = 0 \), so that either or both types are indifferent between issuing or not issuing new equity.

To prove Proposition 3, we first establish a preliminary lemma:

**Lemma 3:**  
1. If \( \Gamma_E > R_G \), then \( \psi_L(\Gamma_E, \Gamma_0) \geq 0 \) implies \( \psi_H(\Gamma_E, \Gamma_0) > 0 \)
2. If \( \Gamma_E < R_G \), then \( \psi_L(\Gamma_E, \Gamma_0) \leq 0 \) implies \( \psi_H(\Gamma_E, \Gamma_0) < 0 \)
3. \( \psi_L(\Gamma_E, \Gamma_0) = 0 \) implies \( \psi_H(\Gamma_E, \Gamma_0) = (\Gamma_E - R_G) E \left(1 - \lambda \right) \frac{\Gamma_H}{\Gamma_E} \), so that if \( \Gamma_E = R_G \), \( \psi_H(\Gamma_E, \Gamma_0) = 0 \)
Proof of lemma 3: This is straightforward using (24).

We now proceed to prove proposition 3: Lemma 3 implies that

$$\psi_H(\Gamma_E, \Gamma_0) = (\Gamma_E - R_G) E (1 - \lambda) \frac{\Gamma_H}{\Gamma_E}.$$ 

As a result, if $\psi_H > 0$, we have $\Gamma_E - R_G > 0$, but then the equilibrium beliefs are such that $\Gamma_0 = \Gamma_L$, and expression (23) tells us that $\psi_L$ is the sum of two positive terms, a contradiction. If instead, $\psi_H < 0$, then, symmetrically, this implies $\Gamma_E - R_G < 0$, and equilibrium beliefs are such that $\Gamma_E = \Gamma_L$, so that expression (23) is the sum of two negative terms, again contradicting $\psi_L = 0$. This establishes the proposition.

Lemma 4: The return on equity-capital for respectively $H$-banks and $L$-banks is given by

$$\Gamma_H = R_G + \frac{\rho_H - c}{\kappa} \text{ and } \Gamma_L = R_G + \frac{\rho_L - c}{\kappa},$$

where

$$\rho_L(\rho_H) = \rho_H \beta - Z, \text{ and } Z = (1 - \beta) v \frac{\mu}{2} \left( \frac{w + E}{\kappa} \right).$$

Proof of Lemma 4: The expression for the return on equity-capital for
H-banks is obvious. As for L-banks, they generate an average return of

$$\Gamma_L = \frac{\int_{p_L}^{p^*} \Gamma_L(p)dp}{p^* - p_L}$$

where,

$$\Gamma_L(p) = R_G + \frac{\rho_H - (1 - p)(1 - \beta)v - c}{\kappa}.$$ 

So that,

$$\Gamma_L = R_G + \frac{\rho_H - c - (1 - \beta)v(1 - \frac{p^* + p_L}{2})}{\kappa}$$

or, after substituting for $p_L$ and $p^*$,

$$\Gamma_L = R_G + \frac{\rho_H\beta - c - (1 - \beta)v\frac{1}{2}\left(\frac{w + \kappa E}{\kappa}\right)}{\kappa}.$$ 

Writing $Z = (1 - \beta)v\frac{1}{2}\left(\frac{w + \kappa E}{\kappa}\right)$, $\rho_L(\rho_H) = \rho_H\beta - Z$ and rearranging we obtain the desired expression for the the return on equity-capital for L-banks.

**Lemma 5:** An H-bank manager is better off issuing equity $E > 0$ than issuing no equity if and only if,

$$\frac{\Gamma_E(w + E) - ER_G}{\Gamma_E} [\lambda \Gamma_E + (1 - \lambda)\Gamma_H] \geq [\lambda \Gamma_0 + (1 - \lambda)\Gamma_H] w. \quad (25)$$

An L-bank manager is better off issuing equity $E > 0$ than issuing no equity
if and only if,
\[ \Gamma_E(w + E) - E R_G \geq \Gamma_0 w. \quad (26) \]

**Proof of Lemma 5:** An \( H \)-bank owner-manager making a new equity issue \( E \) and holding on to her shares until date \( t = 2 \) gets an expected return \((1 - \alpha)\Gamma_H(w + E)\). The same manager making a new equity issue \( E \) and selling her shares at date \( t = 1 \), before loan returns are realized and the bank’s type is revealed gets \((1 - \alpha)\Gamma_E(w + E)\). Substituting for the value of \( \alpha \) in (11) and rearranging, the manager’s payoff can be rewritten as:

\[
V_H(E) = \frac{\Gamma_E(w + E) - E R_G}{\Gamma_E} [\lambda \Gamma_E + (1 - \lambda) \Gamma_H] \quad (27)
\]

Similarly, the manager’s expected payoff from issuing no equity is given by \( \lambda \Gamma_0 + (1 - \lambda) \Gamma_H w \). Comparing these payoffs, we obtain condition (12).

Under the same market expectations, an \( L \)-bank manager issuing equity \( E > 0 \) gets an expected return \( \Gamma_E(w + E) - E R_G \), and a payoff \( \Gamma_0 w \) when issuing no equity. Comparing these payoffs we obtain condition (13).

**Lemma 6:** The return on equity-capital \( \Gamma_E \) expected by investors in equilibrium is given by

\[
\Gamma_E = R_G + \frac{\delta_H(\hat{R}(p), E) \rho_H + \delta_L(\hat{R}(p), E) \rho_L - c}{\kappa} \quad (28)
\]

**Proof of Lemma 6:** Given the observed bank actions \( \hat{R}(p) \) and \( E \),
investors’ updated beliefs about the bank’s type are:

\[ \Gamma_E = \delta_H(\hat{R}(p), E) \Gamma_H + \delta_L(\hat{R}(p), E) \Gamma_L \]

Substituting for \( \Gamma_H \) and \( \Gamma_L \) and rearranging we obtain the desired expression.

**Proof. proposition 4:** For the pooling equilibrium with \( E = 0 \) to exist we must have

\[
\frac{\Gamma^0_L(w + E) - E \cdot R_G(0)}{\Gamma^0_L} \leq \left[ \lambda \Gamma^0_L + (1 - \lambda) \Gamma^0_H \right] w \quad \text{(29)}
\]

and

\[
\Gamma^0_L(w + E) - ER_G(0) \leq \Gamma^0 w \quad \text{(30)}
\]

where,

\[
\Gamma^0 = \bar{\Gamma} = \mu \Gamma^0_L + (1 - \mu) \Gamma^0_H,
\]

\[
\Gamma^0_H = R_G(0) + \frac{\rho_H(0) - c}{\kappa}
\]

and

\[
\Gamma^0_L = R_G(0) + \frac{\rho_L(0) - c}{\kappa}.
\]

It is easy to see that inequalities (29) and (30) both hold when \( \Gamma^0_L \leq R_G(0) \).

Thus, a sufficient condition for this pooling equilibrium to exist is that

\[
c + Z_0 \geq \rho_H(0)\beta, \quad \text{(31)}
\]
where

\[ Z_0 = (1 - \beta)w \frac{\mu w}{2\kappa}. \]

Consider next the pooling equilibrium with \( E^H = E^L = \overline{E} \). Again, for this equilibrium to exist we must have

\[
\frac{\Gamma_{\overline{E}}(w + \overline{E}) - \overline{E} \cdot R_G(\overline{E})}{\Gamma_{\overline{E}}} \left[ \lambda \Gamma_{\overline{E}} + (1 - \lambda)\Gamma_H^{\overline{E}} \right] \geq \left[ \lambda \Gamma_L^{\overline{E}} + (1 - \lambda)\Gamma_H^{\overline{E}} \right] w \tag{32}
\]

and

\[
\Gamma_{\overline{E}}(w + \overline{E}) - \overline{E} R_G(\overline{E}) \geq \Gamma_L^{\overline{E}} w \tag{33}
\]

where,

\[
\Gamma_{\overline{E}} = \bar{\Gamma} = \mu \Gamma_L^{\overline{E}} + (1 - \mu)\Gamma_H^{\overline{E}},
\]

\[
\Gamma_H^{\overline{E}} = R_G(\overline{E}) + \frac{\rho_H(\overline{E}) - c}{\kappa}
\]

and

\[
\Gamma_L^{\overline{E}} = R_G(\overline{E}) + \frac{\rho_L(\overline{E}) - c}{\kappa}.
\]

Note that inequalities (32) and (33) both hold when \( \Gamma_{\overline{E}} \geq R_G(\overline{E}) \). This latter inequality in turn is equivalent to

\[
c \leq (1 - \mu)\rho_H(\overline{E}) + \mu \rho_L(\overline{E})
\]

or,

\[
c + Z_{\overline{E}} \leq (1 - \mu + \mu \beta)\rho_H(\overline{E}),
\]

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where
\[ Z_E = (1 - \beta)v\mu\frac{w + E}{\kappa}. \]

Thus, a sufficient condition for the pooling equilibrium with \( E^H = E^L = E \) to exist is that
\[ c + Z_E \leq (1 - \mu + \mu\beta)\rho_H(E) \tag{34} \]

In sum, if (31) holds then the pooling equilibrium with \( E = 0 \) exists. If, however, (31) does not hold, but
\[ \beta < \beta \equiv \frac{(1 - \mu + \mu\beta)\rho_H(E)}{\rho_H(0)} \]

or,
\[ A3 : \beta < \frac{(1 - \mu)\rho_H(E)}{\rho_H(0) - \mu\rho_H(E)} \]

then (34) holds and the pooling equilibrium with \( E = E \) exists. Indeed, we then have
\[ c + Z_E \leq \rho_H(0)\beta \leq (1 - \mu + \mu\beta)\rho_H(E). \]

Thus, when condition \( A3 \) holds a pooling equilibrium always exists. What is more, both pooling equilibria (with respectively \( E = 0 \) and \( E = E \)) coexist when
\[ A4 : \rho_H(0)\beta < \min\{c + (1 - \beta)v\frac{hw}{2\kappa}; (1 - \mu + \mu\beta)\rho_H(E)\} \tag{35} \]
**Proof. proposition 5:** We use Cramer’s rule to prove the different comparative statics results in the equilibrium where \( R_G > 1 \). Define first the functions \( \Phi(R_G, \rho_H) \) and \( \Omega(R_G, \rho_H) \) from equations (7) and (9) so that both have zero on the RHS:

\[
\Phi(R_G, \rho_H) \equiv R_G + \rho_H \frac{V}{v} - \left(1 - \frac{w + E}{\kappa}\right)(V - v) - v = 0 \quad (36)
\]

\[
\Omega(R_G, \rho_H) \equiv \frac{1}{2R_G} + \rho_H \frac{1}{v} + \frac{E}{\kappa} + w \left(\frac{1}{\kappa} - 1\right) + G - 1 = 0
\]

Then:

\[
\begin{bmatrix}
1 & \frac{V}{v} \\
-\frac{1}{2R_G} & \frac{1}{v}
\end{bmatrix}
\begin{bmatrix}
dR_G \\
d\rho_H
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{\partial \Phi}{\partial G} \\
-\frac{\partial \Omega}{\partial G}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
-\frac{\partial \Phi}{\partial G} \\
-\frac{\partial \Omega}{\partial G}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]

Straightforward computation then lead to

\[
\frac{dR_G}{dG} = \frac{2R_G^2 V}{2R_G^2 + V} > 0,
\]

\[
\frac{d\rho_H}{dG} = -v \frac{2R_G^2}{2R_G^2 + V} < 0
\]
and
\[
\frac{d(R_G + \rho_H)}{dG} = \frac{2R_G^2}{2R_G^2 + V} (V - v) > 0.
\]
The effect on \(p^*\) and \(p^B\) of a change in \(G\) is thus:
\[
\frac{dp^*}{dG} = \frac{2R_G^2}{2R_G^2 + V} > 0
\]
and,
\[
\frac{dp^B}{dG} = \frac{2R_G^2}{2R_G^2 + V} > 0.
\]
In words, there is a shift of bank lending towards safer firms with a constant aggregate amount of bank lending lending. Thus, the effect of an increase in \(G\) is only to reduce the size of the corporate bond market. ■

8.1 An example

We have imposed a number of conditions to guarantee the existence of at least one and possibly multiple pooling equilibria (with maximal bank lending and no junk-bond financing). The reader may wonder whether the set of parameters that satisfy assumptions A1, A2 and A3 is an empty set and whether there are indeed parameter values for which multiple pooling equilibria may exist. For this reason we provide a numerical example below in which multiple pooling equilibria, with maximal bank lending and no junk bond financing, exist.

Let the value of corporate projects be \(V = 2\) and \(v = \frac{1}{2}\). Next, let bank
characteristics be:
\[ \kappa = 0.08, w = 0.01, \overline{E} = 0.01, \mu = 0.1, c = 0.1, \lambda = 0.5, \beta = 0.5. \]
Set government borrowing at \( G = 0.1. \)

Then, straightforward calculations yield:

- \( \frac{E + w}{\kappa} = 0.25, \)
- \( \mathcal{A}(0) = 0.90625, \)
- \( \mathcal{B}(0) = 0.785, \)
- \( \mathcal{A}(0) - \mathcal{B}(0) = 0.12125, \)
- \( \mathcal{A}(\overline{E}) = 0.8125, \)
- \( \mathcal{B}(\overline{E}) = 0.66, \)
- \( \mathcal{A}(\overline{E}) - \mathcal{B}(\overline{E}) = 0.1525. \)

Combining (7) and (9) we obtain a quadratic equation:

\[
-R_G^2 + (\mathcal{A}(E) - \mathcal{B}(E)) VR_G + \frac{V}{2} = 0
\]

and the solution:

- \( R_G(0) = 1.128573961 \) for \( E = 0 \) and,
- \( R_G(\overline{E}) = 1.164061293 \) for \( E = \overline{E}. \)

Replacing for \( R_G \) in either (7) or (9) then yields:
• $\rho_H(0) = 0.17098151$ and therefore $\rho_L(0) = 0.054240755$, and

• $\rho_H(\pi) = 0.115234677$ and therefore $\rho_L(\pi) = 0.026367338$.

For these values of bank spreads, $L$-banks prefer to lend to firms rather than invest in the financial market. This can be seen by observing that the spread $\rho_L(E, p_L(E))$ on the riskiest firm $p_L(E)$ is positive for these numbers:

$$\rho_H(E) - (1 - p_L(E))(1 - \beta)\nu \geq 0 \text{ for } E = 0, \pi.$$

To verify these two inequalities we begin by computing $p_L(0)$ and $p_L(\pi)$ from the formula

$$p_L(E) = 1 - \frac{\rho_H(E)}{v} - \mu\left(\frac{w + E}{\kappa}\right)$$

and get $p_L(0) = 0.87901849$ and $p_L(\pi) = 0.922265323$. From these values we then check that the spread $\rho_L(E, p_L(E))$ on the riskiest firm is positive for $E = 0, \pi$. For the parameter values of our example we get: $\rho_L(0, p_L(0)) = 0.0023995377$ and $\rho_L(0, p_L(0)) = 0.006933669$, which are both positive.

Using these numbers it is also easy to check that assumptions A1 and A2 are satisfied and that

$$p_B(0) < \frac{R_G(0)}{V} \text{ and } p_B(\pi) < \frac{R_G(\pi)}{V}.$$

Assumption A3 also holds since $\beta\rho_H(0) = 0.085490755$ while $(1 - \mu + \mu\beta)\rho_H(\pi) = 0.109472943$. 

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Finally to check the existence of the two pooling equilibria we compute the returns on bank equity:

- $\Gamma^0_H = 2.015842833$,
- $\Gamma^0_L = 0.556583397$,
- $\Gamma_0 = 1.869916889$,
- $\Gamma^E_H = 1.354494752$,
- $\Gamma^E_L = 0.243653023$, and
- $\Gamma^E = 1.243410579$.

For these returns, the pooling equilibrium with $E = 0$ exists, if

\[
\frac{\Gamma^0_L (w + E) - E \cdot R_G}{\Gamma^0_L} \left[ \lambda \Gamma^0_L + (1 - \lambda) \Gamma^0_H \right] \leq \left[ \lambda \Gamma_0 + (1 - \lambda) \Gamma^0_H \right] w
\]

(37)

and

\[
\Gamma^0_L (w + E) - E R_G(0) \leq \Gamma_0 w
\]

(38)

The first condition is satisfied, as

\[
\frac{\Gamma^0_L (w + E) - E \cdot R_G}{\Gamma^0_L} \left[ \lambda \Gamma^0_L + (1 - \lambda) \Gamma^0_H \right] - \left[ \lambda \Gamma_0 + (1 - \lambda) \Gamma^0_H \right] w = -0.019784844.
\]

The second condition is also satisfied, since $\Gamma^0_L - R_G(0) < 0$. 

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Finally, the pooling equilibrium with $E^H = E^L = E$ also exists if

$$
\frac{\Gamma_E(w + E) - E \cdot R_G(E)}{\Gamma_E} \left[ \lambda \Gamma_E + (1 - \lambda) \Gamma_H^E \right] \geq \left[ \lambda \Gamma_L^E + (1 - \lambda) \Gamma_H^E \right] w \tag{39}
$$

and

$$
\Gamma_E(w + E) - ER_G(E) \geq \Gamma_L^E w \tag{40}
$$

The first inequality is satisfied as,

$$
\frac{\Gamma_E(w + E) - E \cdot R_G(E)}{\Gamma_E} \left[ \lambda \Gamma_E + (1 - \lambda) \Gamma_H^E \right] - \left[ \lambda \Gamma_L^E + (1 - \lambda) \Gamma_H^E \right] w = 0.005827725
$$

And the second condition also holds as $\Gamma_E > R_G$.

9 Appendix: Non-binding capital adequacy constraints

In practice, most banks have an equity capital base in excess of the BIS equity capital requirement. If equity capital involves a higher cost of capital than other sources of external funding, the obvious question arises, why banks hold equity capital in excess of regulatory requirements.

The answer seems to be that banks want to maintain a lending capacity to be able to meet unexpected new lending opportunities or to be able to
carry out future loan commitments. We can model this idea by assuming that when a firm debt is to be renegotiated, it involves a complete restructuring with an additional cash injection that we assume equal to \( l_1 \). Obviously, if the firm is successful at time 1, no additional external funds are required.

Assumptions \( A_0 \) and \( A_1 \) have then to be adapted:

Assumption \( A_0 \) stating that the average project has a positive net present value has to be modified as follows:

\[
A_0: \pi_H + \nu \pi_H - (1 - p_1)l_1 > 1 + \omega
\]

For simplicity we assume that banks can only issue equity at time 0, (In practice it is difficult to tap the equity market too often). Then, the bank is able to compute its time 1 capital constraint

\[
\frac{E_1}{L_1} \geq \kappa \geq 0
\]

where \( L_1 \) is the amount of outstanding loans at time 1, (which depends on the expected amount of additional loans, \( \int_{p_1}^{p_1^t}(1-p_1)dp_1 \), the nominal rate on these loans, the expected repayment, \( \pi_H \int p_1 dp_1 \) as well as the expected loan losses \( (L_0 - A)(1-\nu) \int p_1 dp_1 ) \) and \( E_1 \) is the equity capital base at time \( t = 1 \), which includes time zero net profits if we assume a zero dividend at time \( t = 0 \).

Depending on the expected amount of profit, additional loans and loan losses, the binding capital constraint will be the one at time \( t = 1 \) while
it will not be binding at time $t = 0$. Thus, we will observe equity capital slack because of the profitability of making additional loans to good firms in distress. Notice therefore that the slack will depend upon the business cycle since the proportion of good firms in distress, the proportion of repayments and the banks profits themselves depend upon the business cycle.

There is an additional condition that is required for banks to perform their role in the loan renegotiation process, that a loan to a firm in distress is more profitable than a new loan to the average firm:

$$\frac{\pi_H}{l_1} \geq \rho_H$$

This condition which is absolutely natural when we think in terms of the incentives and of the credibility of banks to renegotiate their loans is also interesting as it shows banks benefiting from their captive unlucky borrowers. This condition will always hold for high cash flows $\pi_H$ and low cost overruns $l_1$ and low future expected profitability on new loans $\rho_H$. 
References


Notes

1See, e.g. Berger A.N. and G. F. Udell, p.655 (1998) for a review of the empirical literature on the credit crunch.

2‘Bank spreads’ here refer to the difference between the expected return on a bank loan and the yield on government bonds. These spreads are difficult to measure accurately, as banks do not systematically disclose the precise lending terms on the loans they extend.

3From a conceptual point of view, a common weakness of models which allow for only bank lending without any other direct source of funding for firms is that banks in these models look essentially like non-financial firms, the only difference being that they are subject to capital adequacy requirements.

4However, these options are examined in Bolton and Freixas (2000) for firms with a richer cash-flow structure.

5We assume for convenience that the bank appropriates the entire restructuring value. In other words, the bank is an informational monopoly able to extract the entire continuation value as in Rajan (1992). Of course, banks’ ability to extract this value will be anticipated by borrowers and priced into the ex-ante loan terms.
6We restrict attention to parameter values such that the riskiest firms do not issue junk bonds in equilibrium. As we point out in Bolton and Freixas (2000) this is an option that may be attractive to the riskiest firms for some constellation of parameters.

7In Bolton and Freixas (2000) these fund raising costs were specified exogenously. In contrast, here these costs are endogenized.

8Since in our model, banks do not default this condition is the same even in the absence of deposit insurance. If banks could default with positive probability then we would have to introduce a demand for the payment services associated with deposits, so as to make bank deposits attractive while preserving their option to invest in T-Bills as an investment vehicle that does not generate losses.

9The BIS capital adequacy rules in our highly simplified model are that $\kappa = 0.08$ for standard unsecured loans.

10We justify the existence of an upper bound on $E$ by the following potential incentive problem between bank managers and bank shareholders: if the bank raises an amount superior to $E$ then bank managers may have an incentive to abscond with the money or use it to increase their private benefits. Indeed, the larger is $E$ the greater the private benefits relative to the cost in terms of loss of reputation.

11Note that this objective function is similar to that considered by Myers
and Majluf (1984) but it is not vulnerable to the criticisms voiced against
their specification (see e.g. Dybvig and Zender (1989)).

12 More precisely, in Proposition 3 we establish that only pooling or semi-
separating equilibria exist.

13 We do not consider outcomes where $\rho_H < 0$, since banks always have the
option to invest in securities, which provide a zero spread.

14 There are number of possible equilibrium outcomes in our model. In
some equilibria there is a ‘junk-bond’ financed segment of firms. These are
highly risky firms that prefer bank financing, but are not able to afford the
intermediation cost. In other equilibria $L$-banks do not extend any bank
loans at all, or do not lend to capacity, as they cannot find enough firms
to lend to that provide a sufficiently high return (recall that $L$-banks are
only able to generate a restructuring return $\beta v$ (with $1 > \beta$) where $H$-banks
generate a return $v$). Although these equilibria are of interest, we shall not
analyze them, as they lead to a parallel, possibly more cumbersome analysis
of the bank-capital monetary policy transmission channel.

15 This does not mean, however, that riskier firms have to meet higher
contractual repayments $\hat{R}(p)$. In fact, it is easy to show that assumption A1
implies that firms with higher risks will pay higher interest rates. The reason
is simply that firms with higher risks also generate lower expected returns.

16 The partition of firms into three classes, those that are credit rationed,
those that are bank financed, and those that are financed through securities issues has been obtained in earlier models but for different reasons. Holmstrom and Tirole (1997), for example, emphasize the role of collateral, and have firms with more collateral issue securities. Berger and Udell (1991), on the other hand, assume that firms differ in the extent of their private information and obtain that those firms that have a higher level of asymmetric information are credit rationed, those with the least asymmetric information are funded by financial markets (arm length finance) and those in between are funded by banks through monitored finance.

17Note that assumption A1 implies that $p_B^1 > 0$ for $\rho_H > 0$.

18As we have pointed out, firms with $p < p_B$ may be unable to get a bank loan but may possibly be able to get junk-bond financing. The minimum $p$ for which bond financing is available is given by $pV = R_G$. If $\frac{R_G}{V} < p_B^1$ then clearly the segment of risks between $\frac{R_G}{V}$ and $p_B^1$ would be able to issue junk-bonds. We do not consider such equilibria, as they are somewhat of a distraction.

19To see this, note that if $x > (1 - p_L(x))(1 - \beta)v$ then $y > x$ implies that $y > (1 - p_L(y))(1 - \beta)v$. Indeed, replacing $p_L(x)$ by its value, yields, $x > (\frac{\mu}{v} + \mu \frac{w + E}{k})(1 - \beta)v$, which is equivalent to $\beta x > \mu(\frac{w + E}{k})(1 - \beta)v$. Therefore, if this inequality holds for $x$ it must also hold for $y > x$.

20This is always true in our model for $H-$banks, but only true for $L-$banks
under assumption A2. We also explain in appendix B that dynamic considerations may induce banks to keep a small equity-capital ‘cushion’, for inventory-management reasons. This is an additional reason why banks in reality hold equity-capital in excess of 8 percent.

In general one might expect that $H$-banks would make different equity issue decisions $E^H$ than the decisions $E^L$ by $L$-banks. However, as the next section makes clear, in our model only pooling equilibria such that $E^H = E^L = E$ can exist.

Or, to issue less new equity.

Semi-separating equilibria, where one or both of the types randomize over two levels of equity issues $\hat{E} = 0$ and $\hat{E} > 0$, may also exist. To avoid a lengthy digression, we shall not consider semi-separating equilibria here.

This system of equations admits two possible solutions for $R_G(E)$ and $\rho_H(E)$, but only one root $R_G(E)$ is greater than one under assumptions A1 and A2.

Note that our qualitative results and comparative statics analysis do not depend on this refinement in any way. What is important for our analysis is essentially that pooling equilibria do indeed exist and that bank lending may vary with equity-market beliefs.

An interesting observation emerging from this analysis is that it is pos-
sible to observe at the same time a reduction in bank spreads, accompanied by a contraction in aggregate private investment, and an increase in the remuneration of deposits.

The model we have considered here is an oversimplification of reality to the extent that we have ruled out equity financing by firms. In Bolton and Freixas (2000) we allow firms to raise funds in any form they like, including equity. The analysis in that model suggests that another possible effect of a monetary tightening is to induce substitution away from equity financing towards bank lending and bond financing.
Figure 1: Timeline

- $t=0$: Lending terms, Equity decisions, Loan applications, Banks investment choice, Household savings decisions
- $t=1$: Firms returns realized, defaulting firms restructured or liquidated, Bank equity market opens
- $t=2$: Remaining contractual obligations are fulfilled

Figure 2: Bank Lending Equilibrium

- Securities market equilibrium
- Credit market equilibrium

Figure 2:
Figure 3: Multiple Pooling Equilibria

Figure 4: Crowding out effect